PREFACE

The Army Institute for Professional Development (AIPD) administers the consolidated Army Correspondence Course Program (ACCP), which provides high-quality, economical training to its users. The AIPD is accredited by the Accrediting Commission of the Distance Education and Training Council (DETC), the nationally recognized accrediting agency for correspondence institutions.

Accreditation is a process that gives public recognition to educational institutions which meet published standards of quality. The DETC has developed a thorough and careful evaluation system to assure that institutions meet standards of academic and administrative excellence before it awards accreditation.

The many TRADOC service schools and DOD agencies that produce the ACCP materials administered by the AIPD develop them to the DETC standards.

The AIPD is also a charter member of the Interservice Correspondence Exchange (ICE). The ICE brings together representatives from the Army, Navy, Air Force, Marine Corps, and Coast Guard to meet and share ideas on improving distance education.

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INTRODUCTION

This is the first of three subcourses that are an introduction to or refreshers for your knowledge of basic electricity. This reviews the mathematics you need to understand the basic operating principles of guided missile systems and electronic and radar circuits. Covered are algebra, logarithms, trigonometry, and vector algebra.

Supplementary Requirements

There are no supplementary requirements in material or personnel for this subcourse. You will need only this book and will work without supervision.

Credit Hours

Five credit hours will be awarded for the successful completion of this subcourse-a score of at least 75 on the end-of-subcourse examination.

Administrative Instructions

Change Sheets. If a change sheet has been sent to you with this subcourse, be sure you post the changes in the book before starting the subcourse.

Errors on TSC Form 59. Before you begin this subcourse, make sure that the information already typed on your TSC Form 59 (ACCP Examination Response Sheet) is correct. You will find the correct subcourse number and subcourse edition number on the front cover of this book. If any of the information on your TSC Form 59 is incorrect, write to:

The Army Institute for Professional Development (IPD)
US Army Training Support Center
Newport News, VA 23628-0001

A new, correctly filled-out form will be sent to you. Do not correct the form yourself or send it to IPD.

Questions, Changes, Corrections. If you have questions about enrollment or other administrative matters, write to IPD. If a change occurs or a correction needs to be made in your status (name, grade, rank, address, unit of assignment, etc.) notify IPD as soon as possible. These kinds of changes or corrections can be sent along on a separate sheet of paper with your completed TSC Form 59.

Correspondence with IPD. In any correspondence with IPD, always write your name, Social Security Number, and the school code of your enrollment on each page.

Grading and Certification Instructions

When you have completed the subcourse, review any of the material covered that you are not sure of. Then take the end-of-subcourse examination. When you have completed the examination in the book, you must transfer your answers to TSC Form 69. The instructions on the form itself tell you how to mark your answers. Follow the instructions carefully.

Once you have transferred your answers to the TSC Form 59, fold the form as it was folded when sent to you. Do not staple or mutilate this form! Place the form in the self-addressed envelope provided and mail it to IPD. No postage is needed. TSC Form 59 is the only material that you are required to return to IPD. If you return it as soon as you have completed this subcourse, you will get your next subcourse sooner.

Grading. The highest score possible on the end-of-subcourse examination is 100. The grade structure for all ACCP subcourses is given below:

Superior	95-100
Excellent	85-94
Satisfactory	75-84
Unsatisfactory	0-74

Your TSC Form 59 will be machine graded, and you will be notified of the results. Your grade on the examination will be your grade for the subcourse. No credit is given for grades below satisfactory (75).

Certificates. When you have completed the subcourse successfully, IPD will send you a subcourse completion certificate. Keep it with your other personal copies of personnel material. Subcourse completion certificates can be used to support accreditation and other personnel actions.

*** * * IMPORTANT NOTICE * * ***

THE PASSING SCORE FOR ALL ACCP MATERIAL IS NOW 70%.

PLEASE DISREGARD ALL REFERENCES TO THE 75% REQUIREMENT.

Lesson 1
ALGEBRA

Task. The skills and knowledge taught in this subcourse are common to all missile repairer tasks.

Objectives. When you have completed this lesson you should be able to correctly solve equations using algebraic principles and involving exponents, radicals, and complex numbers.

Conditions. You will have this subcourse book and work without supervision.

Standard. You must score at least 75 on the end-of-subcourse examination that covers this lesson and lessons 2, 3, and 4 (answer 23 of the 30 questions correctly).

Algebra extends the scope of arithmetic by introducing the concept of negative values and the use of letters for numbers. Letters used to represent quantities are called literal numbers. An algebraic expression is any combination of signs, numerals, and literal numbers. For example, a + b and x/2y are algebraic expressions. Signs are used to indicate whether numbers are positive (+) or negative (-), or they may indicate operations to be performed, such as, add (+) or subtract (-).

THE ARITHMETIC

Multiplication of two algebraic quantities need not be indicated by a sign. Just a position of terms can indicate multiplication. Thus, a X b or a . b may be written ab. In the expression ab, a and b are known as factors of the product. Each factor of a product is known as the coefficient of the other factor(s).

The absolute value of a number refers to its magnitude, regardless of the sign preceding it. Numbers not preceded by a sign are assumed to be positive. For example, 8 is the absolute value of both +8 and -8. To designate absolute value, write $|8|$.

An exponent is a number or letter which indicates the power to which a quantity (called the base) is to be raised. It means the number of times the quantity is multiplied by itself. Thus the expression e^x is read e to the x power, signifying that e is multiplied by itself x times.

Any arithmetical or literal number, or the product or quotient of the numbers, is called a term. For example, 4, x, 25b, and x/y are terms.

Terms which have identical literal parts are called similar or like terms, while those with unlike literal numbers are known as unlike terms. Thus 7a and a are like terms, while 7a and $7a^2$, or 8a, and 8b are unlike terms.

Like terms may be added to or subtracted from each other. For example, 4x may be added to 5x to produce 9x, or $10y^2$ - $3y^2$ equals $7y^2$, since the literal numbers are the same in each case. The sum of numbers such as 4y and $5m^2$ must be indicated as 4y + $5m^2$ since y and m^2 are unlike terms.

Addition

To add two numbers with the same sign, add their absolute values and write the common sign before the sum.

+6 added to +3 equals +9
-6 added to -3 equals -9

To add two numbers with opposite signs, take the difference of their absolute values and write the sign of the larger absolute value.

+6 added to -3 equals +3
-6 added to +3 equals -3

Subtraction

To subtract a quantity from another, change the sign of the quantity *to be subtracted*, then add the quantities following the rules of addition.

(-6) - (-3) becomes (-6) + (+3) = -3
(-3) - (-6) becomes (-3) + (+6) = +3
(+6) - (-3) becomes (+6) + (+3) = +9
(+6) - (+3) becomes (+6) + (-3) = +3

Polynomials

An algebraic expression containing two or more terms joined together by a plus (+) or minus (-) sign is called a polynomial. Thus, the expressions a + b - c and ab - ac are polynomials. A polynomial with only two terms is known as a binomial; one with three terms is known as a trinomial; and a single term such as a, b, $10a^2b$ is a monomial.

Polynomials are added and subtracted the same way single terms are.

Addition	*Subtraction*
$4a + 2b + 3c$	$3m^2 - 6n^2$
$+6a + b + 7c$	$-3m^2 - 8n^2$
$10a + 3b + 10c$	$6m^2 + 2n^2$

Signs of Grouping

Certain symbols such as parentheses ( ), brackets [ ] and braces { } are used to group several quantities which are affected by the same operation and must be treated as a single quantity. For example, in the expression $(4a^2 - 3ab) - (3a^2 + 2ab)$, the entire quantity of $(3a^2 + 2ab)$ is subtracted from the first term. In removing a symbol or grouping that is preceded by a minus sign, change the signs of all terms included by the symbol.

$-(3a^2 + 4ab - x)$ becomes $- 3a^2 - 4ab + x$

When one symbol of grouping appears within another, it is best to remove one set of grouping symbols at a time starting with the innermost symbol first.

$-[2am - (2a^2 + 5am) + a^2]$

Remove inside parentheses to obtain

$-[2am - 2a^2 - 5am + a^2]$

Remove brackets to obtain

$- 2am + 2a^2 + 5am - a^2$

Combine terms to obtain

$3am + a^2$

Multiplication

Multiplication of polynomials is similar to the arithmetical multiplication of numbers consisting of several digits. Thus $(2c^2 - 3c - 5)(- 4c + 7)$ may be solved as follows:

$$\begin{array}{r} 2c^2 - 3c - 5 \\ \times \quad -4c + 7 \\ \hline 14c^2 - 21c - 35 \\ -8c^3 + 12c^2 + 20c \\ \hline -8c^3 + 26c^2 - c - 35 \end{array}$$

When multiplying, the sign, the exponent, and the coefficient must be considered.

Sign. The product of two terms with like signs is positive. The product of two terms with unlike terms is negative.

$(+a) \times (+b) = ab$
$(-a) \times (-b) = ab$
$(-a) \times (+b) = -ab$

Be careful to keep the proper sign when multiplying a long series of terms.

$(-a) \times (-b) \times (-c) = -abc$

Exponent. The exponent of any letter in the product is the sum of the exponents of the factors with the same base.

$a^2 \times a^3 = a^{(2+3)} = a^5$
$a^2 \times b^3 \times a^4 \times b^2 = a^{(2+4)}\, b^{(3+2)} = a^6\, b^5$

Coefficient. The arithmetical coefficient of the product is the product of the absolute values of the coefficients of the terms being multiplied.

$6x^2 \times 2x^5 = 12x^7$
$6X^2 \times 2y^5 = 12X^2y^5$

Division

Division of one polynomial by another is similar to long division in arithmetic. One difference is that the dividend, divisor, and remainder (if there is one) must be arranged in order of ascending or descending powers of some letter.

$$\frac{30c^4 + 3 - 82c^2 = 5c = 11c^3}{3c^2 - 4 + 2c}$$

Solution:

$$
\begin{array}{r}
10c^2 - 3c - 12 \\
3c^2 + 2c - 4 \overline{)\,30c^4 + 11c^3 - 82c^2 - 5c + 3} \\
\underline{30c^4 + 20c^3 - 40c^2} \\
-\ 9c^3 - 42c^2 - 5c \\
\underline{-\ 9c^3 - 6c^2 + 12c} \\
-\ 36c^2 - 17c + 3 \\
\underline{-\ 36c^2 - 24c + 48} \\
7c - 45 \text{ (remainder)}
\end{array}
$$

In the division of polynomials, the sign, exponent, and coefficient must again be taken into consideration.

Sign. The quotient of two positive or two negative quantities is positive; the quotient of a positive and negative quantity is negative.

$$(+a) \div (+b) = +\frac{a}{b}$$

$$(-a) \div (-b) = +\frac{a}{b}$$

$$(-a) \div (+b) = -\frac{a}{b}$$

Exponent. The exponent of any letter in the quotient is the difference of the exponents of the factors with the same base.

$$\frac{x^4}{x^2} = x^{(4-2)} = x^2$$

In division, there is the possibility of obtaining zero and negative exponents. Any quantity with a zero exponent is equal to one.

$$\frac{a^5}{a^5} = a^{(5-5)} = a^0 = 1$$

Any quantity with a negative exponent is equal to the reciprocal of that quantity with the corresponding positive exponent.

$$a^{-3} = \frac{1}{a^3} \text{ or } \frac{1}{a^{-3}} = a^3$$

Coefficient. The coefficient of the quotient is the absolute arithmetic value of the dividend divided by the divisor.

$$\frac{4x^2}{2x} = 2x$$

THE MATHEMATICS

Solving Equations

Axioms. An equation is a statement that two quantities are equal. In solving an equation, you have to use axioms (statements accepted as true without proof). The following are a few of the more commonly used axioms plus examples that illustrate how they are used to solve problems.

- If the same number is added to or subtracted from each side of an equation, the result is still an equation.

$$x - 5 = 3$$

Add 5 to each side.

$$\begin{aligned} x - 5 + 5 &= 3 + 5 \\ x &= 8 \end{aligned}$$

- If both sides of an equation are multiplied or divided by the same quantity (not zero), the result is still an equation.

$$5x = 25$$

Divide by 5.

$$\frac{5x}{5} = \frac{25}{5}$$
$$x = 5$$

- If like roots or powers are taken of both sides of an equation, the result is still an equation.

$$\sqrt{x} = 2$$

Square both sides.

$$x = 2^2$$
$$x = 4$$

Forming and Solving Equations. The solving of general equations cannot be explained by any set of rules, for these rules would not hold true in every case. The *most* important single thing to remember is to thoroughly understand what must be translated into mathematical language from the facts or wording of the problem being considered. The items below will help you to understand this.

- Carefully read the problem. Be sure you understand all facts and relationships.

- Determine exactly what you are looking for (the unknown quantity) and designate it by a different letter. If more than one unknown exists, try to represent them in terms of each other.

- Select two expressions based on the facts of the problem that represent the same quantity and place them equal to each other. The equation, thus formed, can then be solved for the unknown.

Example:

The first angle of a given triangle is 40^o less than the second angle. The first angle is greater than the third angle by 10^o. Since the sum of the three angles in any triangle is 180^o, how many degrees does each angle contain?

Solution:

First angle = second angle - 40^o

First angle = third angle + 10^O

Let x = first angle.

Therefore, second angle = $x + 40^O$ and third angle = $x - 10^O$

Select two expressions and place them equal to each other.

$$\begin{aligned} x + (x + 40^O) + (x - 10^O) &= 180^O \\ 3x + 30^O &= 180^O \\ 3x &= 180^O - 30^O \\ 3x &= 150^O \\ x &= 50^O = \text{first angle} \\ 50^O + 40^O &= 90^O = \text{second angle} \\ 50^O + 10^O &= 40^O = \text{third angle} \end{aligned}$$

Factoring. Factoring is the process of finding two or more quantities, each called a factor, whose product is equal to a given quantity. For example, factoring the expression ax + ab - az would produce the expression a(x + b - z) which is still equal in value to the original expression.

Fractions and Fractional Equations. A fraction is an indicated division in which the numerator is the dividend, and the denominator is the divisor. The value or ratio of the fraction is unchanged if both numbers are multiplied or divided by the same number (not zero). Dividing both numerator and denominator by the same number is called reduction to lower terms.

$$\frac{14}{21} = \frac{2 \text{ X } 7}{3 \text{ X } 7} = \frac{2}{3} \qquad \frac{6ax}{9a} = \frac{2 \text{ X } 3ax}{3 \text{ X } 3a} = \frac{2x}{3}$$

Addition and Subtraction. Fractions may be added to, or subtracted from, each other only when they have a common denominator. If the denominators are not alike, you may have to restate the fractions in terms of equivalent fractions with each having what is known as a least common denominator (LCD).

$$\frac{x}{4} + \frac{3x}{5} - \frac{2x}{6}$$

Since the least common denominator is 60, the problem resolves into the following equivalent expression.

$$\frac{15x}{60} + \frac{36x}{60} - \frac{20x}{60} = \frac{15x + 36x - 20x}{60} = \frac{51x - 20x}{60} = \frac{31x}{60}$$

Multiplication. Multiplication of two fractions involves finding the product of both the numerators and the denominators. Frequently, it is possible to further reduce the resulting answer by factoring out common terms and dividing or "cancelling out" these terms.

$$\frac{a^2}{b} \text{ X } \frac{2ac}{d} \text{ X } \frac{1}{2a} = \frac{a^2 \text{ X } 2ac \text{ X } 1}{b \text{ X } d \text{ X } 2a} = \frac{2a^3c}{2abd} = \frac{a^2c}{bd}c$$

Complex fractions. Complex fractions are fractions which contain fractions in both the numerator and denominator. They may be solved as simple division problems.

$$\frac{a\frac{b}{c}}{b\frac{a^2}{c^2}}$$

Solution:

$$a\frac{(b)}{c} \div b\frac{(a^2)}{c} = \frac{ab}{c} \div \frac{a^2b}{c} = \frac{ab}{c} \times \frac{c}{a^2b} = \frac{1}{a}$$

Simultaneous Linear Equations. Simultaneous linear equations are two or more equations that contain only first powers of the unknown quantities and no products of unknowns. They are also equations that have only certain common values of the unknowns. You can solve for the unknowns obtained by graphing, by elimination by addition or subtraction, by elimination by substitution, or by using determinants. Of these methods, elimination by subtraction is the most common. Solve for e and i in the following two equations.

$$2e + 10i = 25 \quad (1)$$
$$5e - 8i = 46 \quad (2)$$

Solution:

Multiply equation (1) by 4, and get

$$8e + 40i = 100. \quad (3)$$

Multiply equation (2) by 5, and get

$$25e - 40i = 230. \quad (4)$$

Add equation (4) to equation (3).

$$\begin{aligned} 8e + 40i &= 100 \quad (3)\\ \underline{25e - 40i} &\underline{= 230} \quad (4)\\ 33e + 0 &= 330 \quad (5) \end{aligned}$$

Divide equation (5) by 33, and get

$$e = 10. \quad (6)$$

Substitute the value of e from equation (6) in either equation (1) or (2)

$$\begin{aligned} 2(10) + 10i &= 25 \quad (1)\\ 20 + 10i &= 25\\ 10i &= 25 - 20\\ i &= \frac{5}{10} = \frac{1}{2} \end{aligned}$$

Therefore, for the given conditions:

$e = 10.$

$i = \frac{1}{2}$

Exponents, Radicals, and Complex Numbers

Exponents. Remember that an exponent is a number or letter multiplied by itself some indicated number of times. Remember, too, that a number raised to the zero power is equal to 1 ($x^O = 1$), and that a negative exponent is the same as the reciprocal of the quantity to the same positive exponent ($x^{-2} = 1/x^2$). Now consider the significance of a fractional exponent. By squaring the quantity $x^{1/2}$, you obtain $(x^{1/2})(x^{1/2})$. By adding the exponents ($x^{1/2 + 1/2}$), as in any other multiplication of exponential numbers, we now obtain x1 or x as the result. By reversing these procedures, you can state that the square root of x is $x^{1/2}$. Also, by the same reasoning, ($x^{1/3}$) is the cube root of x and so on. In the expression $x^{2/3}$ (which reads x to two-thirds power), you can say that it is equivalent to the cube root of x^2 or it can be expressed mathematically as $\sqrt[3]{-x^2}$

Radicals. Sometimes it is necessary to simplify an expression involving radicals (square roots, cube roots, etc.) without changing its value. It may be possible to divide the quantity under the radical (the radicand) into two factors and then take the root of one of the factors (see below).

$$\sqrt{80} = \sqrt{16 \text{ X } 5} = 4\sqrt{5}$$

$$\sqrt[3]{16m} = \sqrt[3]{8 \text{ X } 2m} = 2\sqrt[3]{2m}$$

Should the quantity under the radical be a fraction, multiply both numerator and denominator by a number which will make it possible to extract the root of the denominator.

$$\sqrt{\frac{2}{3}} = \sqrt{\frac{2}{3} \text{ x } \frac{3}{3}} = \sqrt{\frac{6}{9}} = \frac{\sqrt{6}}{\sqrt{9}} = \sqrt{\frac{6}{3}} = \frac{1}{3}\sqrt{6}$$

$$\sqrt[3]{\frac{40}{x^2}} = \sqrt[3]{\frac{40}{x^2} \text{ X } \frac{x}{x}} = \sqrt[3]{\frac{40x}{x^3}} = \sqrt[3]{\frac{8 \text{ X } 5x}{x^3}} = \frac{2\sqrt[3]{5x}}{x^3} = \frac{2}{x}\sqrt[3]{5x}$$

Radicals may be multiplied by other radicals provided they are of the same root. To do this, multiply the coefficients to get the new coefficient and multiply the radicands to get the new radicand of the product.

$$4\sqrt{x} + 3\sqrt{x} = 7\sqrt{x}$$

$$5\sqrt[3]{y} - 3\sqrt[3]{y} = 2\sqrt[3]{y}$$

Radicals may be multiplied by other radicals provided they are of the same root. To do this, multiply the coefficients to get the new coefficient and multiply the radicands to get the new radicand of the product.

$$2\sqrt{a} \text{ x } 3\sqrt{b} = 6\sqrt{ab}$$

Division of radicals is the reverse of multiplication. Coefficient is divided by coefficient, and radicand is divided by radicand.

$$\frac{6\sqrt{5}}{3\sqrt{6}} = 2\sqrt{\frac{5}{6}}$$

It is usually better to eliminate the fraction under the radical. The value $2\sqrt{5/6}$ may be further changed as follows.

$$2\sqrt{\frac{5}{6}} = 2\sqrt{\frac{5}{6} \text{ x } \frac{6}{6}} = 2\sqrt{\frac{30}{36}} = \frac{2}{6}\sqrt{30} = \frac{1}{3}\sqrt{30}$$

Since fractions involving radicals in the denominator are normally clumsy to manipulate when solving algebraic expressions, it is often better to rearrange the fraction so that a whole number, a fraction, or mixed number will appear in the denominator only. To eliminate a radical from the denominator of a binomial expression, use the process known as rationalization of the denominator.

In this instance, the numerator and denominator are multiplied by the conjugate of the denominator. (A conjugate is the same expression with the sign reversed between the terms.) To illustrate this procedure, consider the expression

$$\frac{5}{3-\sqrt{2}}.$$

Multiply $\frac{5}{3-\sqrt{2}}$ by

$$\frac{3+\sqrt{2}}{3+\sqrt{2}}$$

where

$$3+\sqrt{2}$$

is the conjugate of

$$3-\sqrt{2}$$

Therefore,

$$\frac{5}{3-\sqrt{2}} \text{ x } \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{5\ (3+\sqrt{2})}{(3-\sqrt{2})\ (3+\sqrt{2})}$$

$$= \frac{15+5\sqrt{2}}{9-3\sqrt{2}+3\sqrt{2}-2}$$

$$= \frac{15+5\sqrt{2}}{9-2}$$

$$= \frac{15+5\sqrt{2}}{7}$$

Complex Numbers. Thus far you have dealt only with the roots of positive numbers. Consider now negative quantities appearing under radicals, such as √-x or √-3. There is no quantity that when squared, will produce -x or -3; therefore, the square roots of negative numbers are designated as "imaginary numbers." The expressions √-x and √-3 can each be visualized as (√-1) (√x) and (√-1) (√3). The term √-1 has been arbitrarily designated "i," an imaginary number equal to √-1. (In electrical work and in this subcourse, the symbol "j" is used since the symbol "i" is used to designate current.) The term √-x can now be further expressed as j√x, and the term √-3 can be expressed as j√3. The following relationship for j (or i) will prove helpful when solving problems involving imaginary numbers (figure 1-1).

$$\sqrt{-1} = j$$
$$-1 = j^2$$
$$-\sqrt{-1} = j^3$$

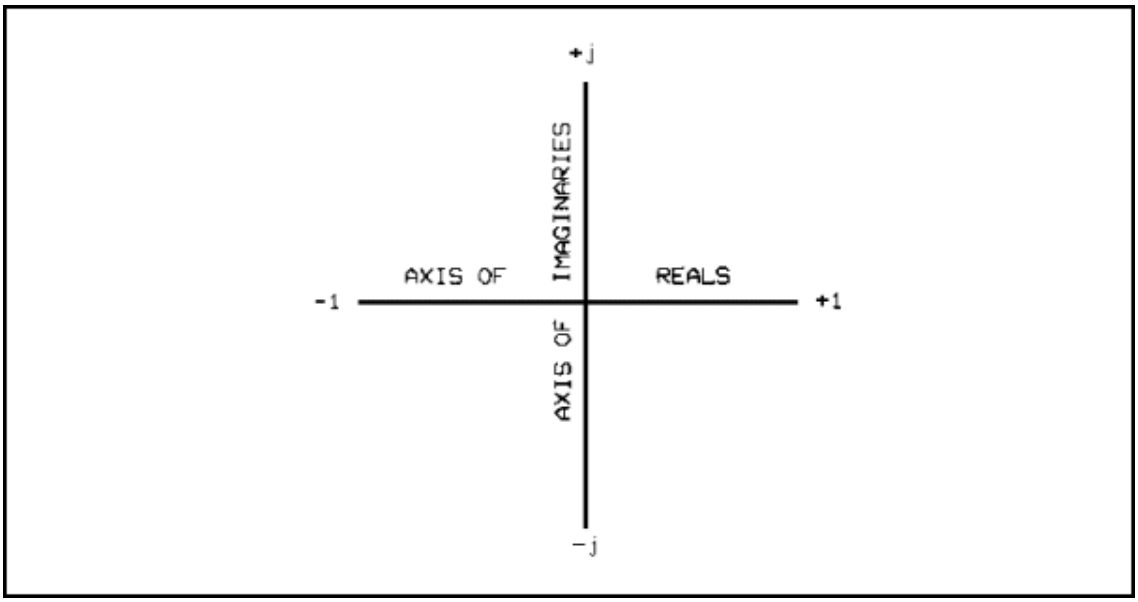

Figure 1-1. Plotting Real and Imaginary Numbers.

The quantity √-1 is usually referred to as the "j operator" and is frequently used in the solution of alternating current problems. Real and imaginary quantities can be graphically represented by four positions of a unit vector as in figure 1-1. Positive real numbers are plotted horizontally to the right, and negative real numbers are plotted horizontally to the left. Positive imaginary numbers are plotted vertically above the horizontal axis, and negative imaginary numbers are plotted vertically below the horizontal axis.

A complex number is the sum or difference of a real quantity and an imaginary quantity. Thus, 5 + j3 and 2 - j2 are complex numbers. Complex numbers can be easily combined if you combine the real and imaginary portions separately.

Add (5 + j3) and (2 -j2).

$$\begin{array}{r} 5 + j3 \\ +\,2 - j2 \\ \hline 7 + j \end{array}$$

Subtract (5 + j3) from (6-j4).

$$\begin{array}{r} 6 - j4 \\ -5 - j3 \\ \hline 1 - j7 \end{array}$$

Multiplication of complex numbers is identical to the procedure for the multiplication of binomials; however, whenever the term j^2 appears in the final result it is replaced by its actual value of -1.

$$\begin{array}{l} (5 + j3)(2 - j2) \\ 5 + j3 \\ 2 - j2 \\ \hline 10 + j6 \\ \quad - j10 - j^2 6 \\ \hline 10 - j4 - j^2 6 \end{array}$$

Therefore,

$$\begin{aligned} 10 - j4 - j^2 6 &= 10 - j4 - (-1)6 \\ &= 10 - j4 + 6 \\ &= 16 - j4 \end{aligned}$$

Division of complex numbers entails the rationalization of the denominator and division of the real number you get in the denominator into the numerator. Consider the expression

$(5 + j3) \square (2 - j2)$.

You first rationalize the denominator by multiplying both numerator and denominator by the conjugate of the denominator.

$$\frac{5 + j3}{2 - j2} \text{ X } \frac{2 + j2}{2 + j2} = \frac{10 + j16 + j^2 6}{4 - j^2 4}$$

Simplify the new expression by replacing j2 with -1 and combining the terms as follows.

$$\begin{aligned} \frac{10 + j16 + j^2 6}{4 - j^2 4} &= \frac{10 + j16 + (-1)6}{4 - (-)4} \\ &= \frac{10 + j16 - 6}{4 + 4} \\ &= \frac{4 + j16}{8} \\ &= 0.5 + j2 \end{aligned}$$

Quadratic Equations

Equations assuming the general form $ax^2 + bx + c = 0$ are known as second degree or quadratic equations in x. (The degree is established by the highest exponent value of x, which in this instance is x^2.) A quadratic equation may be solved by several methods, including graphing, completing the square, and factoring, or by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a, b and c are respectively the coefficients of x^2, x, and the constant term. Before applying the formula, arrange the equation into the general form $ax^2 + bx + c = 0$. The following example illustrates the use of the quadratic formula.

Solve for x in the equation

$x + 3x^2 = 10.$

Rearrange terms.

$3x^2 + 8x - 10 = 0$

The coefficients for use in the formula are

$a = 3,$

$b = 8,$

$c = -10.$

Substituting in the formula and solving, you get

$$x = \frac{-8 \pm \sqrt{64 + 120}}{6}$$

$$= \frac{-8 \pm \sqrt{184}}{6}$$

$$= \frac{-8 \pm 13.56}{6}$$

$$= \frac{-21.56}{6} \text{ or } \frac{5.56}{6}$$

$$= -3.59 \text{ or } 0.93.$$

The quantity $b^2 - 4ac$, which appears under the radical in the formula, is called the discriminant. It indicates the type of roots. If $b^2 - 4ac$ is positive, there are two real and unequal roots; if $b^2 - 4ac$ is negative, the roots are imaginary and unequal; if $b^2 - 4ac$ equals zero, the roots are real and equal.

REVIEW EXERCISES

1. Which of the following is the proper notation of the absolute value of a number?

   a. 5.

   b. -5.

   c. (5).

   d. $|5|$.

2. Which of the following numbers is the one having an exponent?

   a. 2x.

   b. 2/x.

   c. x^2.

   d. (4x).

3. What is the right answer to the following equation?

   $6 - 3 + 6 + 4 =$

   a. 16.

   b. 13.

   c. -13.

   d. -16.

4. What is the correct answer to the following problem?

   Subtract:

   $3x^2 - 6y^2$

   $-3x^2 - 8y^2$

   a. $-14y^2$.

   b. $6x^2 + 2y^2$.

   c. $6x^2 - 2y^2$.

   d. $6x^2 - 14y^2$.

5. Which is the number surrounded by braces?

a. $|5|$.

b. (7).

c. [10].

d. {9}.

6. What is the correct answer to the following equation?

$(6x3)\ (2x4) + 3x7 =$

a. $15x^7$.

b. $15x^{12}$.

c. $12x^8$.

d. 15.

7. Which of the following is the correct expression of a quantity having a negative exponent?

$x^{-3} =$

a. x^3.

b. $1/x^3$.

c. $x^3/1$.

d. $\sqrt{x^3}$.

8. Which of the following is the correct answer to the problem?

Solve for x.

$x^2 = 8 - 4$

a. = 4.

b. = 2.

c. = 8.

d. = -2.

9. Which of the following is the correct answer to the problem?

Factor $ax^2 + bx = cy - dy^2$

a. $y(ax + b) = x(c - dy)$.

b. $ax^3 + b = c - dy^3$.

c. $x(ax + b) = y(c - dy)$.

d. $ab + x^3 = cd - y^3$.

10. Which of the following is the correct expression for i or j?

a. $\sqrt{1.}$

b. j2.

c. j3.

d. $\sqrt{-1}$

Recheck your answers to the Review Exercises. When you are satisfied that you have answered every question to the best of your ability, check your answers against the Exercise Solutions. If you missed three or more questions, you should retake the entire lesson, paying particular attention to the areas in which your answers were incorrect.

Lesson 1 Review Exercise Solutions

REVIEW EXERCISES SOLUTIONS

1. Which of the following is the proper notation of the absolute value of a number?

   a. 5.

   b. -5.

   c. (5).

   d. $|5|$.

2. Which of the following numbers is the one having an exponent?

   a. 2x.

   b. 2/x.

   c. x^2.

   d. (4x).

3. What is the right answer to the following equation?

   $6 - 3 + 6 + 4 =$

   a. 16.

   b. 13.

   c. -13.

   d. -16.

4. What is the correct answer to the following problem?

   Subtract:

   $3x^2 - 6y^2$

   $-3X^2 - 8y^2$

   a. $-14y^2$.

   b. $6x^2 + 2y^2$.

   c. $6x^2 - 2y^2$.

   d. $6x^2 - 14y^2$.

5. Which is the number surrounded by braces?

   a. $|5|$.

   b. (7).

   c. [10].

   d. {9}.

6. What is the correct answer to the following equation?

$(6x3)\ (2x4) + 3x7 =$

a. $15x^7$.

b. $15x^{12}$.

c. $12x^8$.

d. 15.

7. Which of the following is the correct expression of a quantity having a negative exponent?

$x^{-3} =$

a. x^3.

b. $1/x^3$.

c. $x^3/1$.

d. $\sqrt{x^3}$.

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Solve for x.

$x^2 = 8 - 4$

a. = 4.

b. = 2.

c. = 8.

d. = -2.

9. Which of the following is the correct answer to the problem?

Factor $ax^2 + bx = cy - dy^2$

a. $y(ax + b) = x(c - dy)$.

b. $ax^3 + b = c - dy^3$.

c. $x(ax + b) = y(c - dy)$.

d. $ab + x^3 = cd - y^3$.

10. Which of the following is the correct expression for i or j?

a.. $\sqrt{1}$.

b. j2.

c. j3.

d. $\sqrt{-1}$

Lesson 1

Lesson 2
LOGARITHMS

Task. Skills and knowledge taught in this lesson are common to all missile repairer tasks.

Objectives. When you have completed this lesson, you should be able to describe the parts of a logarithm and use logarithmic procedures to solve problems correctly.

Conditions. You will have the subcourse book and work without supervision.

Standard. You must score at least 75 on the end-of-subcourse examination that covers this lesson and lessons 1, 3, and 4 (answer 23 of the 30 questions correctly).

In lesson 1, you saw that exponential terms with like bases can be multiplied, divided, squared, etc., by keeping their bases and performing simple addition, subtraction, multiplication, or division of their exponents to get the solution (see the following four examples).

$$a^2 \times a^3 = a^{2+3} = a^5$$
$$a^3 \times a^{-2} = a^{3-2} = a$$
$$(a^2)^3 = a^{2+2+2} = a^6$$
$$\sqrt{a^4} = a^{4/2} = a^2$$

Sometimes complicated arithmetical computations can be simplified by writing the numbers as powers of some base. Consider a very simple example in which powers of 2 are used. The first 12 powers of 2 are arranged as shown.

Exponent	1	2	3	4	5	6	7	8	9	10	11	12
Powers of 2	2	4	8	16	32	64	128	256	512	1024	2048	4096

Problem:

Find the product of 16 and 64 using the powers of base 2 chart above.

Solution:

Since $16 \times 64 = 2^4 \times 2^6 = 2^{10}$, you can find the product of 16 and 64 as follows.

Find the corresponding exponents 4 and 6 above 16 and 64 in the chart. The sum of these exponents is 10, and the number that corresponds to the exponent 10 in the table, 1,024, is the product of 16 and 64.

Other examples follow.

Example 1.

$$\begin{aligned} 4096 \div 512 &= 2^{12} \div 2^{9} \\ &= 2^{(12-9)} \\ &= 2^{3} \\ &= 8 \end{aligned}$$

Example 2.

$$\frac{2084 \times 4096 \times 256}{64 \times 512 \times 32 \times 16} = \frac{2^{11} \times 2^{12} \times 2^{8}}{2^{6} \times 2^{9} \times 2^{5} \times 2^{4}}$$

$$= \frac{2^{(11+12+8)}}{2^{(6+9+5+4)}}$$

$$= \frac{2^{31}}{2^{24}}$$

$$= 2^{(31-24)}$$

$$= 2^{7}$$

$$= 128$$

As you can see, you are limited in the choice of numbers that can be manipulated by this short method because the table is small. While it would be possible by advanced mathematics to compute fractional exponents for numbers between those given in the second row of the table above, it is more practical for computational purposes to use a table based on powers of 10 instead of powers of 2.

Exponents used for the computation above are called logarithms. Thus, since 8 = 23, the logarithm of 8 to the base 2 is 3. In abbreviated form this is written as

$\log_2 8 = 3.$

When 10 is used as the base, you have the following.

$$\begin{aligned} \log_{10} 1 &= 0 \text{ since } 1 = 10^{0} \\ \log_{10} 10 &= 1 \text{ since } 10 = 10^{1} \\ \log_{10} 100 &= 2 \text{ since } 100 = 10^{2} \\ \log_{10} 1{,}000 &= 3 \text{ since } 1{,}000 = 10^{3} \\ \log_{10} 0.1 &= 1 \text{ since } 0.1 = \frac{1}{10} 1 = 10^{-1} \\ \log_{10} 0.01 &= 2 \text{ since } 0.01 = \frac{1}{10} 2 = 10^{-2} \end{aligned}$$

The log of any number between 0 and 10 will be a fraction between 0 and 1; the log of any number between 10 and 100 will be a value between 1 and 2, and so forth.

You are now ready to consider a more formal statement concerning logarithms. The logarithm of a quantity is the exponent or the power to which a given number, called the base, must be raised to equal that quantity. If $N = 10^x$, the exponent x is the log of N to the base 10.

TERMINOLOGY

Systems

Two numbers have been selected as bases, resulting in two systems of logarithms. One base, 2.718, usually indicated by the letter (e), is used in the "natural" logarithm system. The other base is 10 and is used in "common" system of logarithms. In the common system, the base 10 is usually omitted in the logarithmic expression. Thus, $\log_{10} 1{,}000 = 3$ is usually written log 1,000 = 3. In the natural system, the base e is usually written in the logarithmic expression. The logarithms discussed in this subcourse are "common" logarithms.

Parts of a Logarithm

For numbers not exact powers of 10, the logarithm consists of two parts: a whole number, called the "characteristic", and the decimal part, called the "mantissa." For example, the logarithm of 595 is 2.7745. In this instance, the characteristic is found by inspection while the mantissa is obtained from log-arithmic tables.

Characteristic. The characteristic of a logarithm can be determined by the following rules.

- The characteristic of the logarithm of a number greater than 10 is positive and is one less than the number of digits to the left of the decimal point.

Number	*Characteristic*
5	0
23	1
567.8	2
8432.29	3

- The characteristic of the logarithm of a number less than one is negative and is equal to one more than the number of zeros immediately to the right of the decimal point.

Number	*Characteristic*
.532	-1
.034	-2
.00509	-3

Mantissa. The mantissa of a logarithm is found from a table of logarithms. Numbers which have the same sequence of digits, but differ only in the location of the decimal point, have the same mantissa.

Number	*Mantissa*
595	.7745
59.5	.7745
.595	.7745

PROCEDURES

Finding the Logarithm of a Number

Table 2-1 at the end of the lesson is a complete, four place logarithm table. The first column in the table contains the first two digits of numbers whose mantissas are given in the tables, and the top row contains the third digit. To find the logarithm of a number such as 58.4, first determine the characteristic by inspection. In this instance it is 1 (one less than the number of digits to the left of the decimal point). Now look in the table to determine the mantissa. Remember that the mantissa is the decimal portion of the logarithm and, although it is customary to omit the decimal point in the construction of tables, it must be put in when writing the complete logarithm. Referring to table 2-1, look down the left-hand column for the first two digits, find 58; then go across to the column labeled 4, find the mantissa: .7664. The logarithm for 58.4 is therefore:

log 58.4 = 1.7664.

To find the logarithm of a number with more than three digits, use the process called interpolation. To illustrate this process, determine the logarithm of 5,956. The mantissa for 5,956 is not listed in the table; however, the mantissas for 5,950 and 5,960 are listed. (The mantissa for 5,950 is the same for 595, etc.) Since 5,956 lies between 5,950 and 5,960, its mantissa must lie between the mantissas for these two numbers.

Arrange the numbers in tabular form.

Numbers	*Characteristic*	*Mantissa*
5,960	3.	7,752
5,956	3.	?
5,590	3.	7,745

Since 5,956 lies 6/10 of the way between 5,950 and 5,960, the mantissa must be 6/10 of the way between .7745 and .7752. Since the difference between the two is .0007, and 6/10 of .0007 is .00042, add .00042 to the mantissa of 5,950, and get the result, .77492. The complete logarithm of 5,956, therefore, is 3.77492, rounded off to 3.7749.

Negative Characteristics

When the characteristic of a logarithm is negative, do not put the minus sign in front of the logarithm; it applies only to the characteristic and not the mantissa. Instead, add 10 to the negative characteristic and indicate the subtraction of 10 at the end of the logarithm. Thus the characteristic, if -2, is written:

8. Mantissa -10.

Another method frequently used is to place the negative sign directly above the characteristic. For example:

$\overline{2}$. Mantissa $\overline{4}$. Mantissa

Antilogarithms

The number corresponding to a given logarithm is called the antilogarithm of that number. It is written "antilog" or $\log^{-1}$. To find the antilog, reverse the process for determining the logarithm. To find the antilog of 1.8102, locate the mantissa, .8102, in table 2-1. It is in line with 64 and column 6. Thus, the number corresponding to this mantissa has the digits, 646. To determine the correct position of the decimal point, reverse the procedure used to determine the characteristic. Since the characteristic is 1, there must be two digits to the left of the decimal point. The correct answer, therefore, is 64.6. In some instances, the exact mantissa cannot be found in the tables. You must then use the process of interpolation.

Problem:

Find the antilog of $\bar{3}.7690$.

Solution:

Again referring to table 2-1, you find that .769 lies between the mantissa .7686 (587) and .7694 (588). The difference between the mantissas .7686 and .7694 is .0008. The difference between .769 (the given mantissa) and .7686 is .0004. Find the number corresponding to .7690 by taking .0004/.0008 of 1 or .5 and adding it to 587 and get 5875. Since the given characteristic was $\bar{3}$, you know that there must be two zeros to the decimal point in front of the first digit. Therefore, the correct answer is 0.005875.

Computations with Logarithms

To compute using logarithms, the general rules described for the manipulation of exponents apply.

Multiplication. To multiply two numbers, add their logarithms and find the antilog of the result.

6,952 X 437	=	?
Log 6,952	=	3.8421
Log 437	=	2.6405 (sum)
		6.4826
antilog 6.4826	=	3,038,000

Actual multiplication of 6,952 x 437 would give the result 3,038,024, or an error of .0008 of one percent. The error is due to the number of places in the logarithm tables. Greater accuracy would result from using 5 or 7 place tables.

Division. To divide two quantities, subtract the logarithm of the divisor from the logarithm of the dividend and find the antilog of the result to obtain the quotient.

6,952 ÷ 437	=	?
log 6,952	=	3.8421
log 437	=	2.6405 (subtract)
		1.2016
antilog 1.2016	=	15.908

Therefore,

$6{,}952 \div 437 = 15.908.$

Powers. To raise a quantity to a power, multiply the logarithm of the quantity by the exponent or the power and find the antilog of the result.

$$\begin{aligned} (5.2)^6 &= ? \\ \log (5.2)^6 &= 6 \log 5.2 \\ 6 \log 5.2 &= 0.7160 \text{ X } 6 \\ &= 4.2960 \\ \text{antilog } 4.296 &= 19{,}768 \end{aligned}$$

Therefore,

$(5.2)^6 = 19{,}768.$

To raise a quantity to a negative power, proceed as follows.

$$\begin{aligned} (45.6)^{-3} &= ? \\ \log (45.6)^{-3} &= 3 \log 45.6 \\ \log 45.6 &= 1.6590 \text{ X } -3 \\ &= -4.9770 \end{aligned}$$

Since logarithm tables list only positive values of the mantissa, change the logarithm above by subtracting from 10.0000 - 10).

$$\begin{array}{r} 10.0000 - 10 \\ 4.9770 \\ \hline 5.0230 - 10 \end{array} \text{ or } 5.0230$$

The antilog of

$\bar{5}.0230 = 0.00001054.$

Therefore,

$(45.6)^{-3} = 0.00001054.$

Roots. To find the root of a quantity, divide the logarithm of the number by the indicated root and find the antilog.

$$\begin{aligned} \sqrt[3]{1.572} &= ? \\ \log \sqrt[3]{1.572} &= \frac{1}{3} \log 1.572 \\ \text{Log } 1.572 &= 0.1965 \\ \frac{0.1965}{3} &= 0.0655 \\ \text{antilog } 0.0655 &= 1.163 \end{aligned}$$

Therefore,

$\sqrt[3]{1.572} = 1.163.$

Sample solution of complex problem. Solve

$$\frac{439 \times 6{,}793 \times \sqrt[4]{8.43}}{(4.6)^3 \times 14.9 \times \sqrt{1.02}}$$

Next, simplify the problem by arranging the terms in the following manner.

Numerator:

	log 439 =	2.6425
	log 6,793 =	3.8321
$\log \sqrt[4]{8.43} = 8.43 \div 4$	=	0.2315 (sum)
		6.7061

Denominator:

$\log (4.6)^3 = 3 \log 4.6$	=	1.9884
	log 14.9 =	1.1732
$\log \sqrt{1.02} = 1.02 \div 2$	=	0.0043 (sum)
		3.1659

Now, subtract the log of the denominator (3.1659) from the log of the numerator (6.7063).

$$\begin{array}{r} 6.7061 \\ -3.1659 \\ \hline 3.5402 \end{array}$$

antilog 3.5402 = 3,470

Therefore,

$$\frac{439 \times 6{,}793 \times \sqrt[4]{8.43}}{(4.6)^3 \times 14.9 \times \sqrt{1.02}} = 3{,}470$$

POWERS OF TEN

Frequently it is desirable to determine quickly and with a reasonable degree of accuracy the result of an expression which contains large or cumbersome numbers. At first glance, the expression

$$\frac{536{,}000{,}000 \times .00625 \times 482}{.000006 \times 6{,}213} \qquad (1)$$

brings visions of tedious multiplication and division by conventional methods. The use of logarithms would shorten the calculations, provided logarithm tables were readily available. Suppose, however, our problem could be reduced to some simple expression such as:

$$\frac{5.4 \text{ X } .6.3 \text{ X } 4.8}{6 \text{ X } 6.2} \quad (2)$$

One could arrive at an approximate but fairly accurate answer by merely performing several simple multiplication and division steps. The solution to (1), gotten by conventional multiplication, is

4,331,509,201 (3)

or approximately

4,332,000,000. (4)

The approximate solution to (2) is

4.32. (5)

Simplification

Ignoring the number of zeros or the placement of the decimal points for a moment, notice the simplicity of the digits appearing in (4) and (5). If a simple method were available for determining the position of the decimal point, we would have a virtual "engineers' shorthand" at our disposal. Such a system, known as the powers of 10, does exist and is based upon the relatively simple rules of exponents described in the preceding portions of this subcourse.

To begin with, the number 536,000,000 can be written

536 X 1,000,000

One million can be expressed (using the principle of exponents) as 10^6. The number can now be written

536 X 10^6.

The value 536, however, can be further reduced to 5.36 x 100, and 100 is equal to 10^2. The number 536,000,000 may now be written

5.36 X 10^2 X 10^6.

Combining the exponents, you get a further simplification,

5.36 X 10) $^{(2+6)}$
or
5.36 X 10^8.

The number .000625 can also be rewritten

6.25 X .0001.

Recalling again the laws of exponents and powers of 10, you can see that .0001 is equivalent to 10-4. The expression now becomes 6.25 x 10^{-4}.

Every number in the original expression can now be rewritten in the form of some small number multiplied by some power of 10.

$$\frac{536{,}000{,}000 \times .000625 \times 482}{.000006 \times 6{,}213} =$$

$$\frac{(5.36 \times 10^{8}) \times (6.25 \times 10^{-4})\ (4.82 \times 10^{2})}{(6 \times 10^{-6}) \times (6.213 \times 10^{3})}$$

Regrouping the terms, you get

$$\frac{(5.36 \times 6.25) \times 4.82)\ (10^{8} \times 10^{-4}\ 10^{2})}{(6 \times 6.213)\ (10^{-6} \times 10^{3})} =$$

$$\frac{(5.36 \times 6.25 \times 4.82)\ (10^{6})}{(6 \times 6.213)\ (10^{-3})} =$$

$$\frac{5.36 \times 6.25 \times 4.82 \times (10^{9})}{6 \times 6.213}$$

You can now perform either an exact or approximate multiplication and division of small numbers. Assuming you are interested only in an approximate answer, you can reduce the fraction as follows: (≅ means "approximately equal to")

$$\frac{5.36 \times 6.25 \times 4.82 \times 10^{9}}{6 \times 6.213} \cong$$

$$\frac{5.4 \times \cancel{6.3}^{1} \times \cancel{4.82}^{.8} \times 10^{9}}{\cancel{6}_{1} \times \cancel{6.2}_{1}} \cong$$

$$5.4 \times 0.8 \times 10^{9} \cong$$

$$4.32 \times 10^{9} \cong$$

Referring again to the exact answer of 4,331,509,201, you find that you are accurate to within 2/10 of 1 percent, normally an acceptable degree of accuracy.

Rules

Two basic rules for converting numbers to powers of 10 can be stated as follows:

1. To express a large number as a small number times a power of 10, move the decimal point left to the desired location and count the number of places to the original decimal point. The number of places moved will be the proper positive exponent of 10.

$$\begin{aligned} 45 &= 4.5 \times 10^{1} \\ 450 &= 4.5 \times 10^{2} \\ 4{,}500 &= 4.5 \times 10^{3} \\ 45{,}632 &= 4.5632 \times 10^{4} \end{aligned}$$

2. To express a decimal as a whole number times a power of 10, move the decimal to the right and count the number of places moved. The number of places moved is the proper negative exponent of 10.

$0.6 = 6 \times 10^{-1}$
$0.06 = 6 \times 10^{-2}$
$0.006 = 6 \times 10^{-3}$
$0.0006 = 6 \times 10^{-4}$

Reciprocals

Frequently in electronics, expressions will be encountered that involve reciprocal quantities (1 over a particular number), such as

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} .$$

$$x_c = \frac{1}{2\pi fc} .$$

Such problems are easily solved by utilizing the powers of 10 system. For example, the quantity

$$\frac{1}{40{,}000 \times 0.00025 \times 0.000125}$$

can be solved as follows:

Convert all quantities in the denominator to their proper power of 10.

$$\frac{1}{(4 \times 10^{4}) \times (2.5 \times 10^{4}) \times (1.25 \times 10^{-4})}$$

Multiply the numbers and combine the exponents to obtain one final power of 10.

$$\frac{1}{12.5 \times 10^{-4}}$$

Divide 1 by 12.5, move 10-4 to the numerator, and reverse the sign of the exponent (remember $1/x^2 = x^{-2}$). You now get

$$.08 \times 10^{4}$$

Moving the decimal point 4 places to the right, you get the answer, 800.

Numerical Prefixes

Throughout your study of electronics you will frequently encounter quantities which are exceptionally large or exceedingly small in value. To make it easier to express various units such as volts, cycles, amperes, and ohms in terms of their absolute values, metric prefixes such as *kilo*, *mega*, *micro*, and *milli* are added to the word. The use of these prefixes or their abbreviations reduces the requirements for writing large numbers because they are based on powers of 10. At the end of this lesson is a list of these prefixes and their abbreviations are in table 2-2. Table 2-3 is a conversion table for multiplying units exponentially. Table 2-4 shows exponential differences between units.

Table 2-1a. Four-Place Common Logarithms of Numbers.

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1178	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4116	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5582	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981

Table 2-1. Four-Place Common Logarithms of Numbers (cont).

N	0	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8181	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9284	9289	9292
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538

Table 2-1. Four-Place Common Logarithms of Numbers (cont).

N	0	1	2	3	4	5	6	7	8	9
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9618	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9947	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

Table 2-2. Abbreviations for Metric Prefixes

Prefix	*Abbreviation*	*Power of ten equivalent*
Centi	c	10^{-2}
Kilo	k	10^{3}
Mega	M	10^{6}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Pico	p	10^{-12}

Table 2-3. Exponential Multiplication Tables.

Multiply	*By*	*To Obtain*
Units	10^{2}	Centiunits
Units	10^{-3}	Kilounits
Units	10^{-6}	Megaunits
Units	10^{6}	Microunits
Units	10^{3}	Milliunits
Centiunits	10^{-2}	Units
Kilounits	10^{3}	Units
Megaunits	10^{6}	Units
Microunits	10^{-6}	Units
Milliunits	10^{-3}	Units
Kilounits	10^{3}	Kilounits
Megaunits	10^{3}	Kilounits
Megaunits	10^{12}	Microunits
Microunits	10^{-12}	Megaunits
Microunits	10^{-3}	Milliunits

Table 2-4. Exponential Differences Between Units.

TO	FROM					
	MEGA	KILO	UNITS	CENTI	MILLI	MICRO
MEGA		3→	←6	8→	9→	12→
KILO	←3		←3	←5	6→	9→
UNITS	←6			←2	3→	6→
CENTI	←8	←5	←2		1→	4→
MILLI	←9	←6	←3	←1		3→
MICRO	←12	←9	←6	←4	←3	

The numbers indicate the number of places, and the arrows indicate that direction in which the decimal point moves. Units represent all fundamental terms of measure, such as, volts, amperes, ohms, etc. Read the chart from left to right only as indicated in the upper left-hand block.

Example: Convert 0.2 amperes to microamperes. Find the block marked units in the left-hand column. Follow the line across until you are under the block marked micro. You read 6→, which indicates the decimal is to be moved six places to the right. Therefore, 0.2 amperes is equivalent to 200,000 microamperes.

Example: To convert megacycles to kilocycles, read on the mega line across and under kilo. There you find 3→; therefore, 50 megacycles would become 50,000 kilocycles.

Lesson 2 Review Exercises

REVIEW EXERCISES

1. What is the answer to the following problem? Convert to base 2 and give the answer in base 2.

   128 x 32 =

   a. 2^{-10}.

   b. 4^{10}.

   c. 2^{12}.

   d. 4^{-10}.

2. Which of the following base numbers do natural logarithms use?

   a. 3.1418.

   b. 10.

   c. 2.

   d. 2.718.

3. Which of the following base numbers do common logarithms use?

   a. 3.1418.

   b. 10.

   c. 2.

   d. 2.718.

4. What is the decimal part of the logarithm called?

   a. Mantissa.

   b. Exponent.

   c. Characteristic.

   d. Reciprocal.

5. What would be the correct number for the "characteristic" for the following number?

   .00509.

   a. -5.

   b. 0.

   c. -3.

   d. +3.

6. What is the correct answer when you multiply the following numbers using logarithms?

   5,280 x 360 =

   a. 19,008.

   b. 1,900,800.

   c. 1,900.

   d. 1,901.

7. What is the antilogarithm of the following number?

   $\bar{6}.4826$

   a. 6,482,600.

   b. 3,380,000.

   c. .000003038.

   d. 3,241,300.

8. What is the correct way to solve the following problem using powers often?

$$x^c = \frac{1}{6.28 \times 1000 \times .00000005}$$

   a. $.3185 \times 10^3$.

   b. 3.185×10^3.

   c. 6,281.

   d. 6,360,000.

9. What would be the correct symbol for the following number, expressed as micro?

   .000159

   a. 159µ.

   b. 1.59µ.

   c. 1.59m.

   d. 159 x 10^{-6}.

10. What will be the correct answer to the following problem in powers of 10?

$$\frac{200 \text{ Kilo}}{100 \text{ Milli}} =$$

   a. 2 X 10^{3}.

   b. 2 X 10^{-3}.

   c. 200.

   d. 2 X 10^{6}.

Recheck your answers to the Review Exercises. When you are satisfied that you have answered every question to the best of your ability, check your answers against the Exercise Solutions. If you missed three or more questions, you should retake the entire lesson, paying particular attention to the areas in which your answers were incorrect.

Lesson 2 Review Exercises Solutions

REVIEW EXERCISES ANSWER KEY

1. What is the answer to the following problem? Convert to base 2 and give the answer in base 2.

   128 x 32 =

   a. 2^{-10}.

   b. 4^{10}.

   c. 2^{12}.

   d. 4^{-10}.

2. Which of the following base numbers do natural logarithms use?

   a. 3.1418.

   b. 10.

   c. 2.

   d. 2.718.

3. Which of the following base numbers do common logarithms use?

   a. 3.1418.

   b. 10.

   c. 2.

   d. 2.718.

4. What is the decimal part of the logarithm called?

   a. Mantissa.

   b. Exponent.

   c. Characteristic.

   d. Reciprocal.

5. What would be the correct number for the "characteristic" for the following number?

   .00509.

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   5,280 x 360 =

   a. 19,008.

   b. 1,900,800.

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8. What is the correct way to solve the following problem using powers of ten?

$$x^c = \frac{1}{6.28 \times 1000 \times .00000005}$$

   a. .3185 x 10^3.

   b. 3.185 x 10^3.

   c. 6,281.

   d. 6,360,000.

9. What would be the correct symbol for the following number, expressed as micro?

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a. 159µ.

b. 1.59µ.

c. 1.59m.

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10. What will be the correct answer to the following problem in powers of 10?

$$\frac{200 \text{ Kilo}}{100 \text{ Milli}} =$$

a. 2 X 10^{3}.

b. 2 X 10^{-3}.

c. 200.

d. 2 X 10^{6}.

Lesson 2

Lesson 3
TRIGONOMETRY

Task. The skills and knowledge taught in this lesson are common to all missile repairer tasks.

Objectives. When you have completed this lesson, you should be able to explain how trigonometric functions are derived and be able to use those functions to solve trigonometric problems correctly.

Conditions. You will have this subcourse book and work without supervision.

Standard. You must score at least 75 on the end-of-subcourse examination that covers this lesson and lessons 1, 2, and 4 (answer 23 of the 30 questions correctly).

The development and operation of the guided or ballistic missile are based, to a large degree, upon the basic principles of trigonometry. The problems of determining the position of the target in relation to the launch point and the calculations incident to sending a missile to some predetermined point in space or to a point on the surface of the earth all involve some form of trigonometry. Furthermore, many electrical problems, when reduced to triangular equivalents, can be easily and quickly solved by use of trigonometry.

Trigonometry is that phase of mathematics which is concerned with the relationships of sides and angles of a triangle to each other. Several special relationships called trigonometric functions hold true in a right triangle. These trigonometric functions will be the subject of discussion in this lesson.

In the right triangle (figure 3-1), for a given condition of angles, the sides of the triangle will always increase or decrease in the same mathematical proportion and the angles will always total 180°. For example, consider the following triangle.

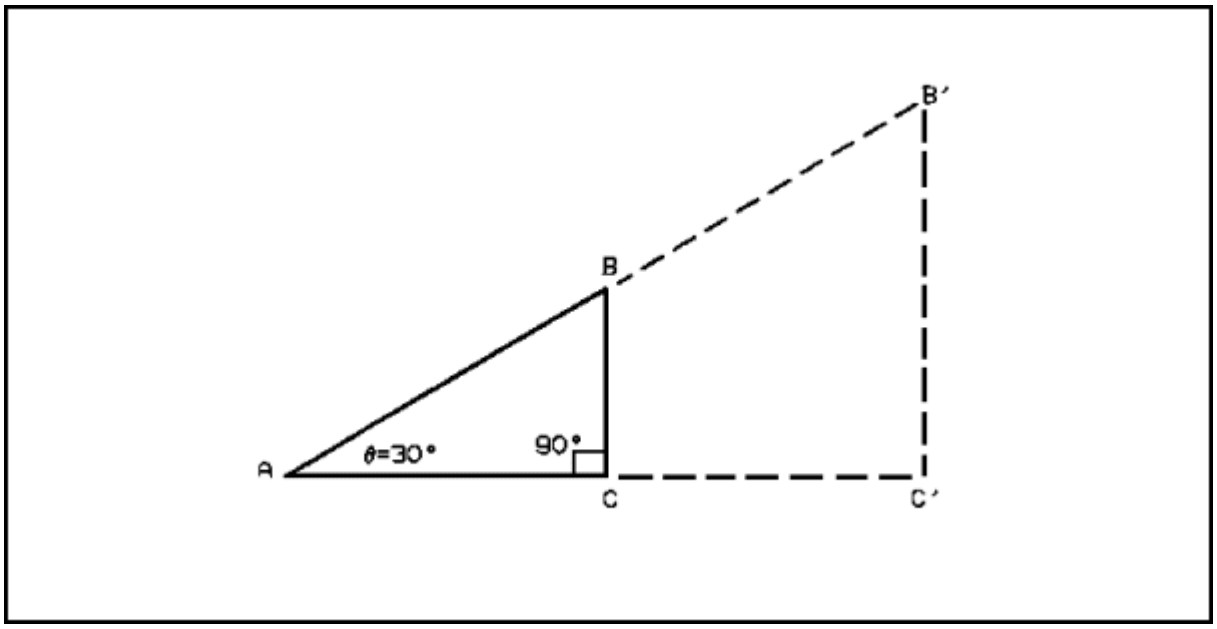

Figure 3-1. Right Triangle Enlarged.

In it, the angle $\theta = 30°$, and the length of the hypotenuse is equal to 1 ft. (θ means "unknown angle," but is often used as angle θ.) Accurate physical measurement of side BC will show that it is equal to 6 in or 0.5 ft, and side AC will be 0.866 ft. Double the length of AB to AB', making it exactly 2 ft long. Actual measurement will show that B'C' is exactly 1.0 ft long, and side AC' is now 1.732 ft long. Stating this relationship in a slightly different manner, you can say that side BC divided by AB is equal to side B'C' divided by AB' or

$$\frac{BC}{AB} = \frac{B'C'}{AB'}$$
$$\frac{0.5}{1} = \frac{1}{2}$$
$$0.5 = 0.5,$$

providing that angle θ did not change from its original value of 30°.

DERIVATION

Bearing in mind the principles established in the previous paragraph, the ratio of any two sides of a right triangle will always produce the same quotient, regardless of the magnitude of the sides, provided the angles remain the same. If each of the possible ratios of the sides of a right triangle were listed and given a specific name, you would have the so-called trigonometric functions. Consider the general right triangle in figure 3-2.

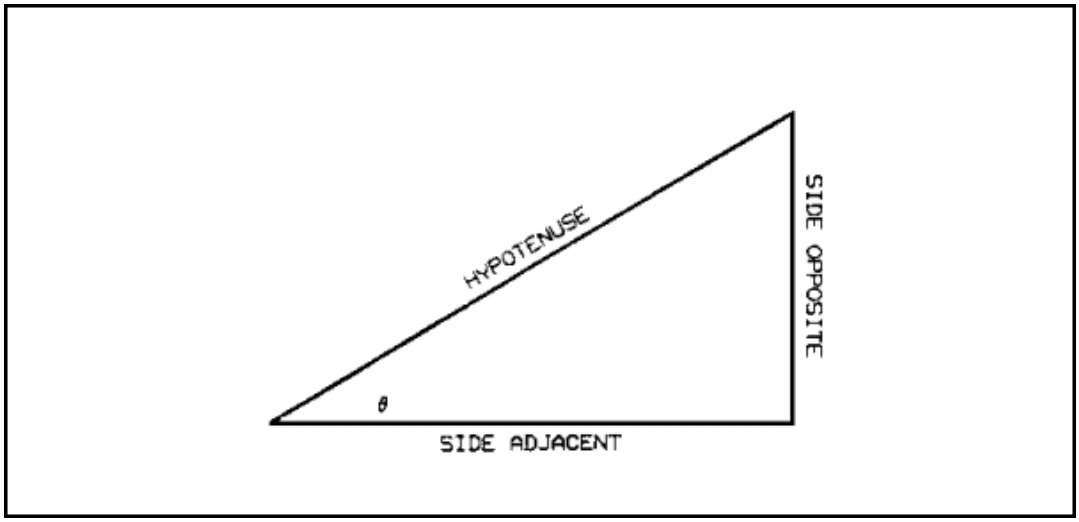

Figure 3-2. Right Triangle.

For the angle θ, there are six possible side ratios which can exist.

1. $\frac{\text{Side opposite}}{\text{Hypotenuse}}$
2. $\frac{\text{Side adjacent}}{\text{Hypotenuse}}$
3. $\frac{\text{Side opposite}}{\text{Side adjacent}}$
4. $\frac{\text{Hypotenuse}}{\text{Side opposite}}$
5. $\frac{\text{Hypotenuse}}{\text{Side adjacent}}$
6. $\frac{\text{Side adjacent}}{\text{Side opposite}}$

Note that the last three ratios are merely the reciprocals of the first three. By relabeling the above ratios as the sine, the cosine, the tangent, the cosecant, the secant, and the cotangent respectively, of the angle é, you can say the following.

1. $\frac{\text{Side opposite}}{\text{Hypotenuse}}$	=	sine of angle θ (written as: sin θ
2. $\frac{\text{Side adjacent}}{\text{Hypotenuse}}$	=	cosine of angle θ (cos θ)
3. $\frac{\text{Side opposite}}{\text{Side adjacent}}$	=	tangent of angle θ (tan θ)
4. $\frac{\text{Hypotenuse}}{\text{Side opposite}}$	=	cosecant of angle θ (csc θ
5. $\frac{\text{Hypotenuse}}{\text{Side adjacent}}$	=	secant of angle θ (sec θ)
6. $\frac{\text{Side adjacent}}{\text{Side opposite}}$	=	cotangent of angle θ (cot θ)

Since the trigonometric functions (sine, cosine, etc.) are merely numerical quotients of a mathematical ratio, you can see that by knowing any two conditions within any of the preceding formulas, you could obtain the third factor. Since the numerical equivalents for the trigonometric functions of any angle from 0 to 90 are listed in tables, it becomes a simple matter to solve mathematical problems involving these ratios. However, to better understand these tables and their uses, construct an abbreviated table based upon the angles 0°, 30°, 45° 60°, and 90°. To simplify your calculations, you need to accept certain axioms from geometry:

- In a 30° - 60° right triangle, the side opposite the 30° angle is equal to one-half of the hypotenuse.
- In a 45°- 45° right triangle, the hypotenuse is equal to the side opposite multiplied by √2.

You also need to simplify your problem by making the hypotenuse a length of one unit. A slight amount of imagination must be exercised in visualizing a right triangle with one of the acute angles as 0° and the other as 90°. However, if you imagine the conditions at "very nearly" 0° and 90°, the problem becomes more realistic.

CONDITION I. Find the value of the trigonometric functions when θ = 0° (figure 3-3).

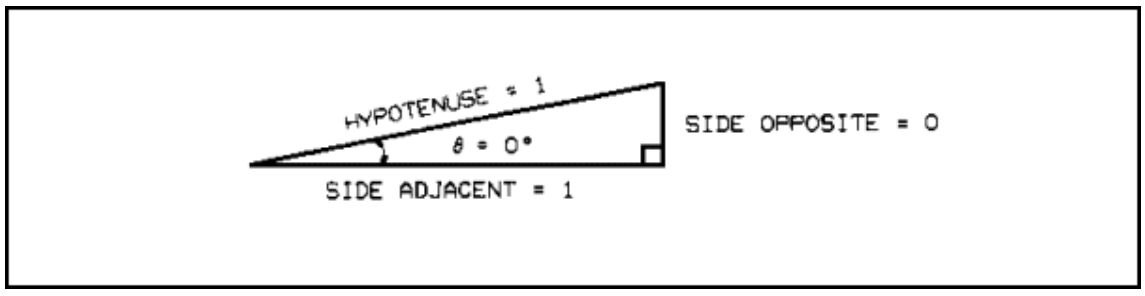

Figure 3-3. Condition I Triangle.

Note. For 0°, the hypotenuse would coincide with the side adjacent making the opposite side equal to zero.

$$\sin 0° = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{0}{1} = 0,$$

$$\cos 0° = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{1}{1} = 1,$$

$$\tan 0° = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{0}{1} = 0,$$

$$\csc 0° = \frac{\text{Hypotenuse}}{\text{Opposite}} = 1 = \text{infinity } (\infty).$$

$$\sec 0° = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{1}{1} = 1,$$

$$\cot 0° = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{1}{0} = \text{infinity } (\infty).$$

CONDITION II. Find the value of the trigonometric functions when $\theta = 30°$ (figure 3-4).

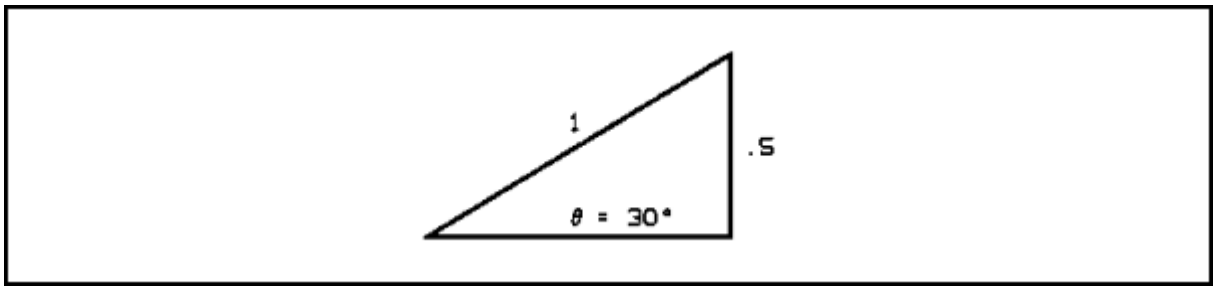

Figure 3-4. Condition II Triangle.

By the Pythagorean theorem,

$$1^2 = (.5)^2 + (\text{adjacent side})^2$$
$$1 = .25 + (\text{adjacent side})^2$$
$$1 = -(.25) = (\text{adjacent side})^2$$
$$\sqrt{.75} = \text{adjacent side}$$
$$0.866 = \text{adjacent side}$$

Therefore,

$$\sin 30° = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{0.5}{1} = 0.5,$$

$$\cos 30° = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{0.866}{1} = 0.866,$$

$$\tan 30° = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{0.5}{0.866} = 0.577$$

$$\csc 30° = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{1}{0.5} = 2$$

$$\sec 30° = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{1}{.866} = 1.155,$$

$$\cot 30° = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{.866}{.500} = 1.732.$$

CONDITION III. Find the value of the trigonometric function when $\theta = 45°$ (figure 3-5).

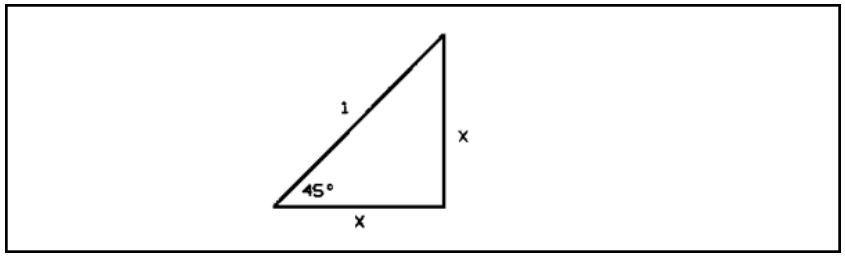

Figure 3-5. Condition III Triangle.

$$x2 + x2 = (1)2$$
$$2x^2 = 1$$
$$x^2 = .5$$
$$x = \sqrt{.5}$$
$$x = 0.707$$

Therefore,

$$\sin 45° = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{0.707}{1} = 0.707,$$

$$\cos 45° = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{0.707}{1} = 0.707,$$

$$\tan 45° = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{0.707}{0.707} = 1$$

$$\csc 45° = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{1}{0.707} = 0.707,$$

$$\sec 45° = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{1}{0.707} = 1.414,$$

$$\cot 45° = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{0.707}{0.707} = 1$$

CONDITION IV. Find the value of the trigonometric functions for $\theta = 60°$

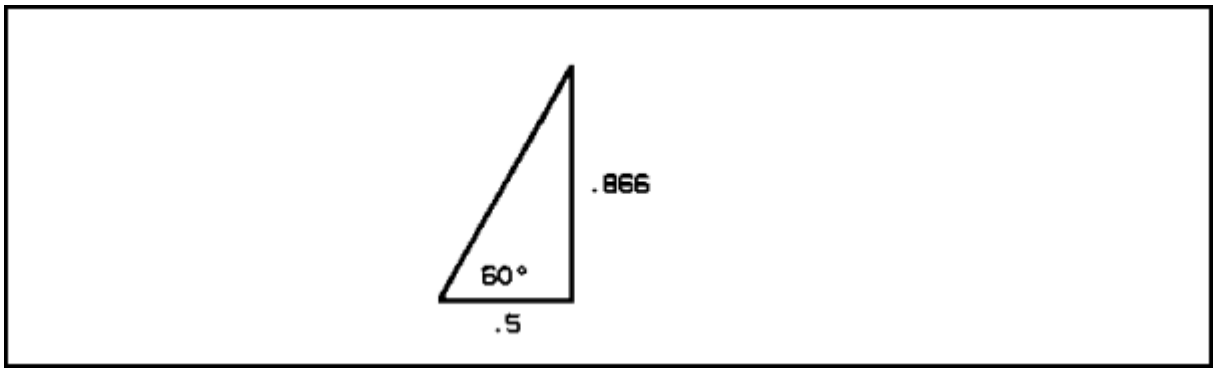

Figure 3-6. Condition IV Triangle.

$$\sin 60° = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{866}{1} = 0.866$$

$$\cos 60° = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{.5}{1} = 0.5$$

$$\tan 60° = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{.866}{.500} = 1.732$$

$$\csc 60° = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{1}{.866} = 1.155$$

$$\sec 60° = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{1}{.5} = 2$$

$$\cot 60° = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{.5}{.866} = 0.577$$

CONDITION V. Find the value of the trigonometric functions when $\theta = 90°$ (figure 3-7).

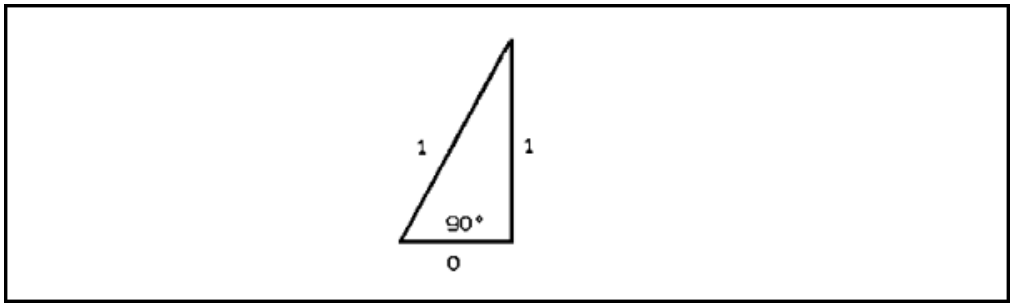

Figure 3-7. Condition V Triangle.

In this instance, the hypotenuse coincides with the opposite side, and the adjacent side is reduced to zero.

Therefore,

$$\sin 90° = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{1}{1} = 1$$

$$\cos 90° = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{0}{1} = 0$$

$$\tan 90° = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{1}{0} = \text{Infinity } (\infty)$$

$$\csc 90° = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{1}{1} = 1$$

$$\sec 90° = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{0}{1} = 0$$

The next step is to list the previously calculated data into tabular form (table 3-1).

Degrees	*Sin*	*Cos*	*Tan*	*Csc*	*Sec*	*Cot*
0	0	1	0	∞	1	∞
30	0.5	0.866	0.577	2	1.155	1.732
45	0.707	0.707	1	1.414	1.414	1
60	0.866	0.5	1.732	1.155	2	0.577
90	1	0	∞	1	∞	0

Table 3-1. Trigonometric Functions.

Except for the intermediate values of angles from 0° to 90°, we have constructed an actual table of trigonometric functions. Note some characteristics of the table. Notice that as the sine increases in value from 0 to 1 (from 0° to 90°), the cosine decreases from 1 to 0 at the same rate. Similar relationships hold true for the other functions. Refer to table 3-8 for a complete table of trigonometric functions.

TRIGONOMETRIC FUNCTIONS

Use

To illustrate the use of trigonometric functions and the use of the trigonometric table developed in the preceding paragraph, follow the solving of the problem illustrated in figure 3-8.

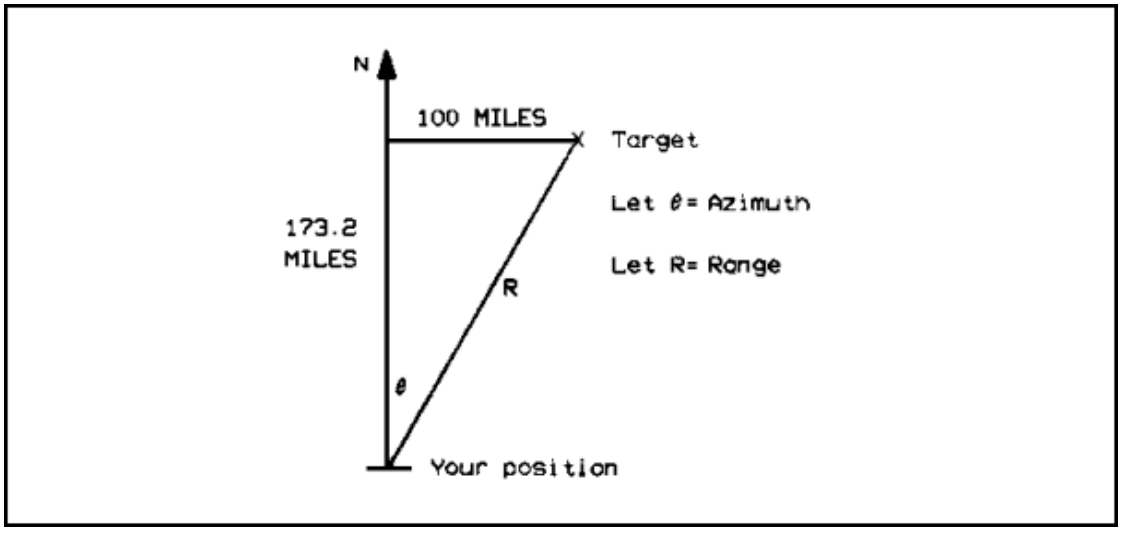

Figure 3-8. Hypothetical Triangle.

Problem:

A target is located 173.2 miles north and 100 miles east of your location. Determine the range and azimuth.

Solution:

First determine θ, remembering that the tangent of

$$\theta = \frac{\text{Side opposite}}{\text{Side adjacent}}$$

$$\tan\theta = \frac{100}{173.2}$$

$$\tan\theta = 0.577$$

Returning to the table developed earlier, you find that 0.577 in the tangent column corresponds to 30°. Therefore,

$$= 30°$$

With θ established, you can now solve for R. Recalling that the sine of an angle is equal to

$$\frac{\text{Side opposite,}}{\text{Hypotenuse}}$$

you can write the following relationship.

$$\sin\theta = \frac{\text{Side opposite}}{\text{Hypotenuse}}$$

$$\sin 30° = \frac{100}{R}$$

Again referring to the table, you find that the sin of 30ø is 0.5. Substituting θ in the formula, you have

$$0.5 = \frac{100}{R},$$

$$0.5R = 100$$

$$R = 200 \text{ miles.}$$

For the given conditions, your answer is now

range (R) = 200 miles, and
azimuth (θ) = 30°.

Quadrants

The functions of angles greater than 90° can be determined by resolving them into an equivalent angle between 0° and 90°. Consider the following coordinate system (figure 3-9) consisting of a Y or vertical axis and an X or horizontal axis. Let all values of X to the right of the origin be positive and all values to the left be negative. Let values of Y above the X axis be positive and those below be negative. Furthermore, label the four quadrants counter-clockwise I through IV.

Consider a unit length vector rotating counterclockwise about the point of origin, starting at the positive X axis at the zero point. At any given instant for the angles 0ø through 360°, the trigonometric functions can be resolved into X and Y components and the unit vector in relation to the angle B between the unit vector and the X axis (figure 3-10).

Summarizing these relationships can be tabulated as in table 3-2.

Specific examples of determining the trigonometric functions of angles greater than 90ø would be as in table 3-3.

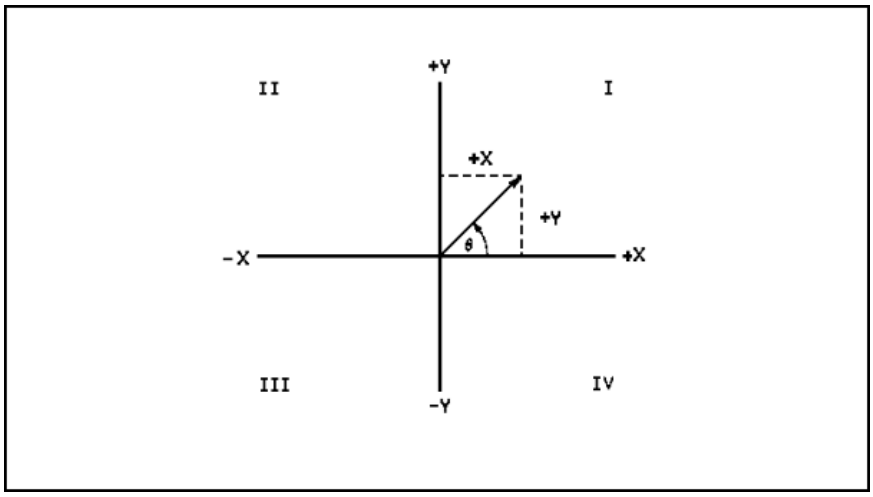

Figure 3-9. Coordinate System.

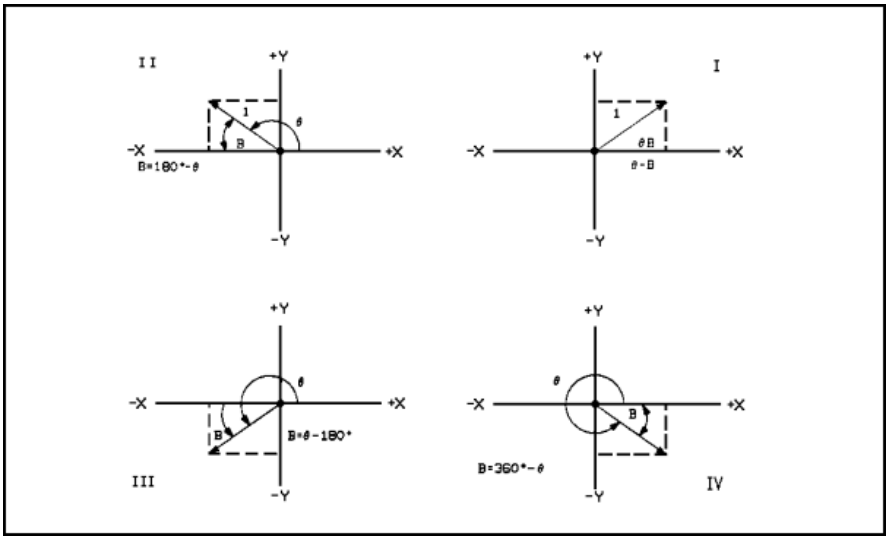

Figure 3-10. Trigonometric Functions as X and Y Components and Unit Vector Relationships. (cont)

Quad I	*Quad II*	*Quad III*	*Quad IV*
$\sin\theta = \frac{+y}{+1}$	$\sin\theta = \frac{+y}{+1}$	$\sin\theta = \frac{-y}{+1}$	$\sin\theta = \frac{-y}{+1}$
$\cos\theta = \frac{+x}{+1}$	$\cos\theta = \frac{-x}{+1}$	$\cos\theta = \frac{-x}{+1}$	$\cos\theta = \frac{+x}{+1}$
$\tan\theta = \frac{+y}{+x}$	$\tan\theta = \frac{+y}{-x}$	$\tan\theta = \frac{-y}{-x}$	$\tan\theta = \frac{-y}{+x}$
$\csc\theta = \frac{+1}{+y}$	$\csc\theta = \frac{+1}{+y}$	$\csc\theta = \frac{+1}{-y}$	$\csc\theta = \frac{+1}{-y}$
$\sec\theta = \frac{+1}{+x}$	$\sec\theta = \frac{+1}{-x}$	$\sec\theta = \frac{+1}{-x}$	$\sec\theta = \frac{+1}{+x}$
$\cot\theta = \frac{+x}{+y}$	$\cot\theta = \frac{-x}{+y}$	$\cot\theta = \frac{-x}{-y}$	$\cot\theta = \frac{+x}{-y}$

Figure 3-10. Trigonometric Functions as X and Y Components and Unit Vector Relationships.

Table 3-2. Signs of Trigonometric Functions.

	Quad I *0° – 90°*	*Quad II* *90° – 180°*	*Quad III* *180° – 270°*	*Quad IV* *270° – 360°*
Sine	+	+	–	–
Cosine	+	–	–	+
Tangent	+	–	+	–
Cosecant	+	+	–	–
Secant	+	–	–	+
Cotangent	+	–	+	–

Table 3-3. Determining Trigonometric Functions, Angles Greater than 90°.

Quad II	sin 95°	=	+ sin (180° – 95°)	=	+ sin 85°
	tan 120°	=	– tan (180° – 120°)	=	– tan 60°
Quad III	cos 200°	=	– cos (200° – 180°)	=	– cos 20°
	cot 250°	=	+ cot (250° – 180°)	=	+ cot 70°
Quad IV	sin 300°	=	– sin (360° – 300°)	=	– sin 60°
	sec 295°	=	+ sec (360° – 295°)	=	+ sec 65°

Radian Measure

In addition to designing the measurement of angles in terms of degrees, it is often necessary or convenient to utilize the system of radian measure (figure 3-11). Specifically, a radian can be defined as "the angle at the center of a circle of radius r that subtends an arc of length r."

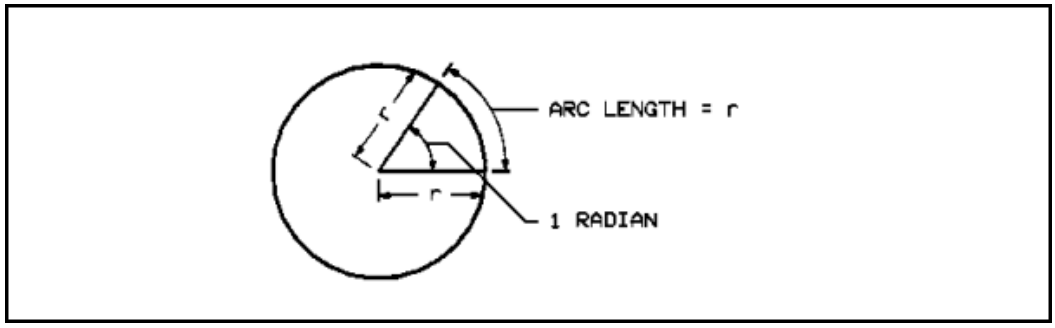

Figure 3-11. Radian.

More precisely, you can state that 360° (one complete circle) is equal to 2 π radians (π = approximately 3.141593).

Therefore,

$$1° = \frac{2\pi \text{ radians}}{360°} = 0.0174533 \text{ radians}$$

or

$$1 \text{ radian} = \frac{360°}{2\pi} = 57.29578 = 57° \ 17' \ 45".$$

A tabular listing of the more common radian and degree equivalent is listed in table 3-4.

Table 3-4. Degree-Radian Equivalents.

Degrees	*Radians*	*Degrees*	*Radians*	*Degrees*	*Radians*
0	0	135	$\frac{3\pi}{4}$	270	$\frac{3\pi}{2}$
30	$\frac{\pi}{6}$	150	$\frac{5\pi}{6}$	300	$\frac{5\pi}{3}$
45	$\frac{\pi}{4}$	180	π	315	$\frac{7\pi}{4}$
60	$\frac{\pi}{3}$	210	$\frac{7\pi}{6}$	330	$\frac{11\pi}{6}$
90	$\frac{\pi}{2}$	225	$\frac{5\pi}{4}$	360	2π
120	$\frac{2\pi}{3}$	240	$\frac{4\pi}{3}$		

Graphic Representation

Frequently, it is necessary to represent the variations of the trigonometric functions in graphic form (figure 3-12). In this subcourse, only the method utilized for developing the sine, cosine, and tangent functions will be discussed; however you would develop the other function curves the same way. Assume that you want to plot the positive or negative value of the sine of an angle as the vertical (Y) ordinate of a graph and the value of θ (the particular angle in question) as the horizontal ordinate of the same graph. The following equation would then summarize our problem.

$Y = \sin \theta$

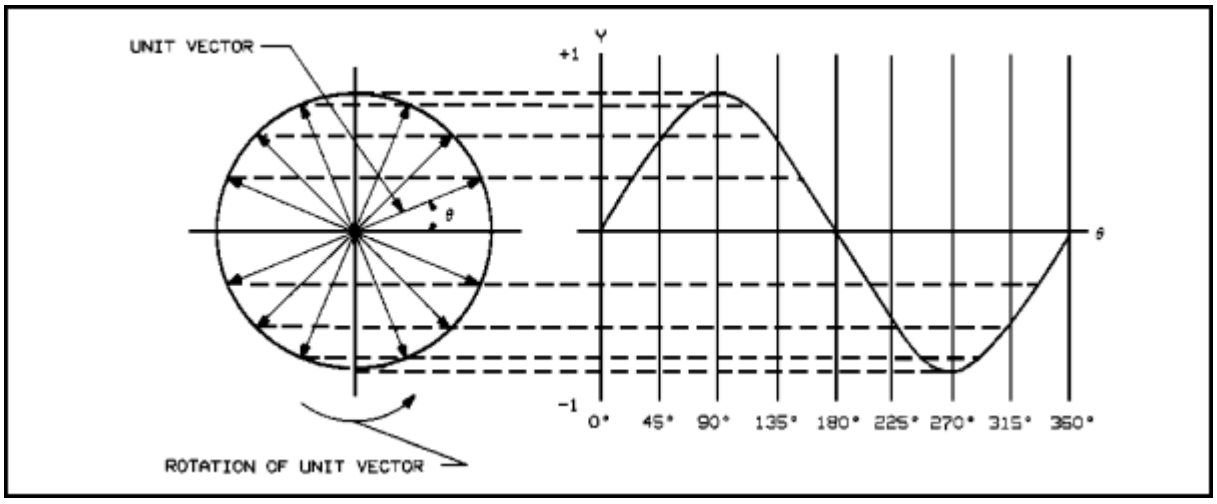

Figure 3-12. Graphic Representation of the Sine Function.

For a moment, consider again the unit vector rotating about the origin of a rectangle coordinate system. As θ increases from 0° to 360°, the sine of the angle will vary between the extremities of +1 and -1 in value. You will draw your gragh immediately to the right and plot the various values of Y or the sine of θ throughout one complete revolution (table 3-5).

Table 3-5. Values of the Sine θ for One Revolution.

θ	*Sin* θ	θ	*Sin* θ	θ	*Sin* θ
0°	0	135°	0.707	270°	– 1
30°	0.5	150°	0.5	300°	– 0.866
45°	0.707	180°	0	315°	– 0.707
60°	0.866	210°	– 0.5	330°	– 0.5
90°	1	225°	– 0.707	360°	0
120°	0.866	240°	– 0.866		

The plot of the cosine of angle θ would follow the same procedure you used in plotting the sine curve. The general expression Y = cos θ would define the plotted curve (figure 3-13 and table 3-6).

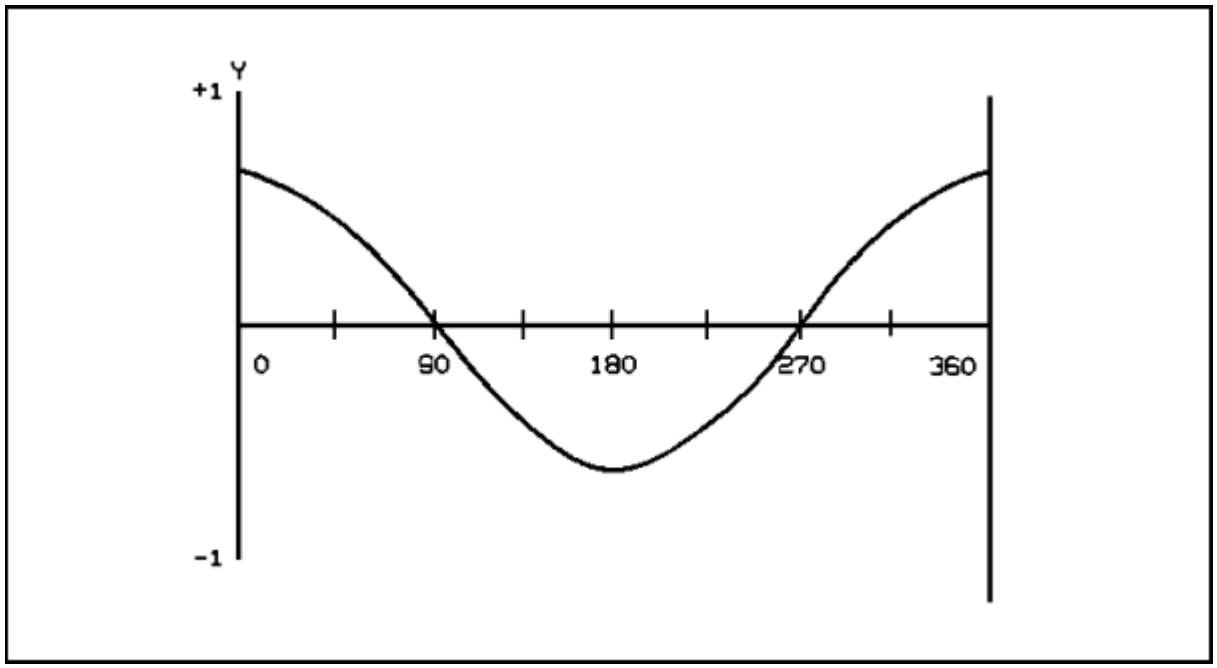

Figure 3-13. Graphic Representation of the Cosine Function.

Table 3-6. Values of the Cosine θ for One Revolution.

θ	Cos θ	θ	Cos θ	θ	Cos θ
0°	1	135°	−0.707	270°	0
30°	0.866	150°	−0.866	300°	0.5
45°	0.707	180°	−1	315°	0.707
60°	0.5	210°	−0.866	330°	0.866
90°	0	225°	−0.707	360°	1
120°	−0.5	240°	−0.5		

The plot of the tangent curve produces a different curve in that it is discontinuous at several points. See figure 3-14 and table 3-7.

Remember, the term *infinity* does not refer to a number of specific magnitude. It more accurately refers to the meaning "approaching an infinite value."

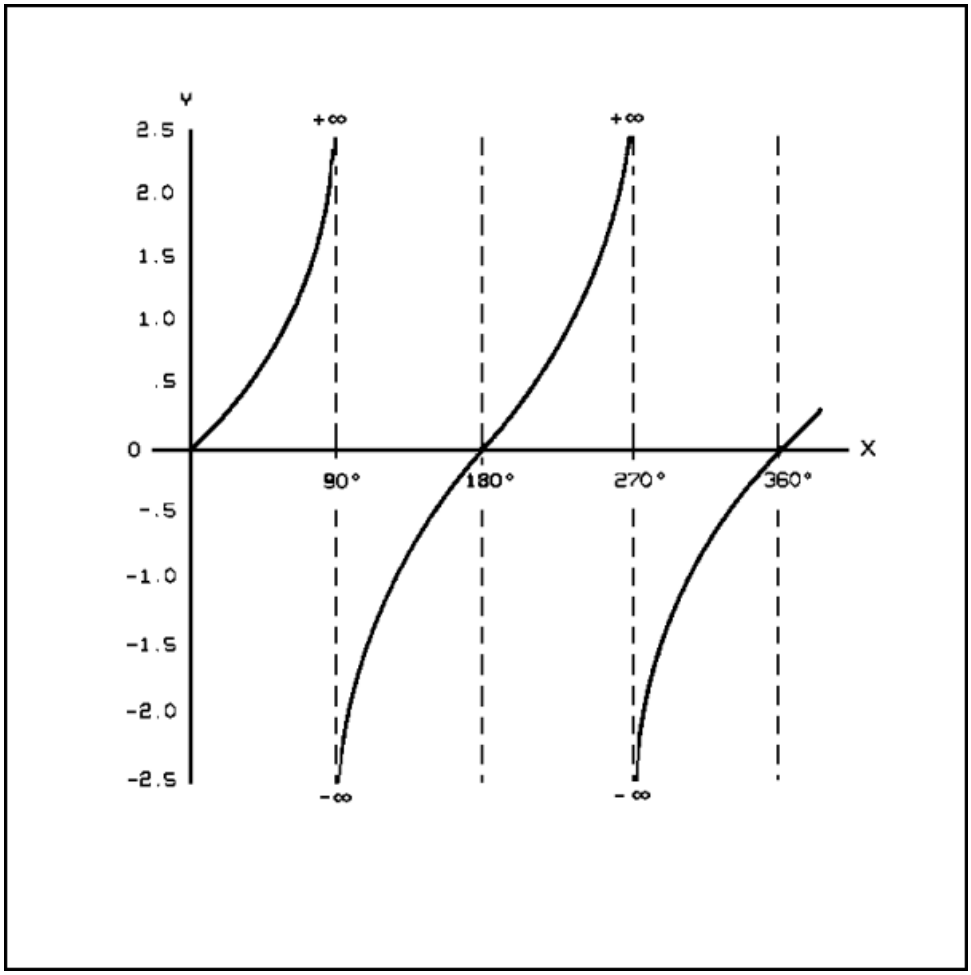

Figure 3-14. Graphic Representation of the Tangent Function.

Table 3-7. Values of the Tangent θ for One Revolution.

θ	$Tan\ \theta$	θ	$Tan\ \theta$	θ	$Tan\ \theta$
0°	0	135°	− 1	270°	∞
30°	0.577	150°	− 0.577	300°	− 1.732
45°	1	180°	0	315°	− 1
60°	1.732	210°	0.577	330°	− 0.577
90°	∞	225°	1	360°	0
120°	− 1.732	240°	1.732		

Table 3-8. Values of Trigonometric Functions.

Angle	*Sin*	*Cos*	*Tan*	*Angle*	*Sin*	*Cos*	*Tan*
0	.0000	1.0000	.0000	46	.719	.695	1.035
1	.0175	.9998	.0175	47	.731	.682	1.072
2	.0349	.9994	.0349	48	.743	.669	1.111
3	.0523	.9986	.0524	49	.755	.656	1.150
4	.0698	.998	.0699	50	.766	.643	1.192
5	.0872	.996	.0875	51	.777	.629	1.235
6	.105	.995	.105	52	.788	.616	1.280
7	.22	.993	.123	53	.799	.602	1.327
8	.139	.990	.141	54	.809	.588	1.376
9	.156	.988	.158	55	.819	.574	1.428
10	.174	.985	.176	56	.829	.559	1.483
11	.191	.982	.194	57	.839	.545	1.540
12	.208	.978	.213	58	.848	.530	1.600
13	.225	.974	.231	59	.857	.515	1.664
14	.242	.970	.249	60	.866	.500	1.732
15	.259	.966	.268	61	.875	.485	1.80
16	.276	.961	.287	62	.883	.469	1.881
17	.292	.956	.396	63	.891	.454	1.963
18	.309	.951	.335	64	.899	.438	2.050
19	.326	.946	.344	65	.906	.423	2.145
20	.342	.940	.364	66	.914	.407	2.246
21	.358	.934	.384	67	.921	.391	2.356
22	.375	.927	.404	68	.927	.375	2.475
23	.391	.921	.424	69	.934	.358	2.605
24	.497	.914	.445	70	.940	.342	2.747
25	.423	.906	.466	71	.946	.326	2.904
26	.438	.899	.488	72	.951	.309	3.078
27	.454	.891	.510	73	.956	.292	3.271
28	.469	.883	.532	74	.961	.276	3.487
29	.485	.875	.554	75	.966	.259	3.732
30	.500	.866	.577	76	.970	.252	4.011
31	.515	.857	.601	77	.974	.225	4.331
32	.530	.848	.625	78	.978	.208	4.705
33	.545	.839	.649	79	.982	.191	5.145
34	.559	.829	.675	80	.985	.74	5.671
35	.574	.819	.700	81	.988	.156	6.314
36	.588	.809	.721	82	.990	.139	7.115
37	.602	.799	.754	83	.993	.122	8.144
38	.616	.788	.781	84	.995	.105	9.514
39	.629	.777	.810	85	.996	.0872	11.44
40	.643	.766	.839	86	.998	.0698	14.30
41	.656	.755	.869	87	.9986	.0523	19.08
42	.669	.743	.900	88	.9994	.0349	28.64
43	.662	.731	.933	89	.9998	.0175	57.29
44	.695	.719	.966	90	1.000	.0000	Infinite
45	.707	.707	1.000				

Lesson 3 Review Exercises

REVIEW EXERCISES

1. Which of the following is the sum of the angles in a triangle?

   a. 90°.

   b. 180°.

   c. 60°.

   d. 360°.

2. On which of the following triangles is the trigonometric relationships based?

   a. Obtuse.

   b. Left.

   c. Right.

   d. Acute.

3. Which of the following is the sine of the right triangle?

   a. $\frac{\text{Side Opposite.}}{\text{Hypotenuse}}$

   b. $\frac{\text{Side Adjacent.}}{\text{Hypotenuse}}$

   c. $\frac{\text{Side Opposite.}}{\text{SideAdjacent}}$

   d. $\frac{\text{Side Adjacent.}}{\text{Side Opposite}}$

4. Which of the following is the tangent of 90°?

   a. Minus one (-1).

   b. One (1).

   c. Zero (0).

   d. Infinity (∞).

5. If the target is located 400 yards north and 300 yards east, what is the range and azimuth?

   a. 500 at 36.87°.

   b. 300 at 62.80°.

   c. 400 at 3.687°.

   d. 700 at 90°.

6. Vectors located in the first quadrant (I), and having an angle less than 90° would be of what polarity?

   a. Part negative.

   b. Part positive.

   c. All negative.

   d. All positive.

7. How many radians are in a circle?

   a. π/2 radians.

   b. 2 π radians.

   c. π radians.

   d. 2 radians.

8. How many degrees are in one radian?

   a. 5.729°.

   b. 6.283°.

   c. 57.295°.

   d. 31.40°.

Lesson 3 Review Exercises Solutions

REVIEW EXERCISES SOLUTIONS

1. Which of the following is the sum of the angles in a triangle?

   a. 90°.

   b. 180°.

   c. 60°.

   d. 360°.

2. On which of the following triangles is the trigonometric relationships based?

   a. Obtuse.

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   c. Right.

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   b. $\frac{\text{Side Adjacent.}}{\text{Hypotenuse}}$

   c. $\frac{\text{Side Opposite.}}{\text{SideAdjacent}}$

   d. $\frac{\text{Side Adjacent.}}{\text{Side Opposite}}$

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   b. One (1).

   c. Zero (0).

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   c. 400 at 3.687°.

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   c. All negative.

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7. How many radians are in a circle?

   a. π/2 radians.

   b. 2 π radians.

   c. π radians.

   d. 2 radians.

8. How many degrees are in one radian?

   a. 5.729°.

   b. 6.283°.

   c. 57.295°.

   d. 31.40°.

Lesson 4
VECTOR ALGEBRA

Task. The skills and knowledge taught in this lesson are common to all missile repairer tasks.

Objectives. When you have finished this lesson, you should be able to explain what vector algebra is and be able to use it to solve problems correctly.

Conditions. You will have the subcourse book and work without supervision.

Standard. You must score at least 75 on the end-of-subcourse examination that covers this lesson and lessons 1, 2, and 3 (answer 23 of the 30 questions correctly).

In the first three lessons, you studied basic algebra, logarithms, and trigonometry. They are all necessary preparation for this lesson because vector algebra combines their properties.

VECTOR QUANTITIES

Some quantities have magnitude only; others have both magnitude and direction. Quantities with magnitude only are known as scalar quantities, while those with both magnitude and direction are known as vector quantities. Forces, velocity, and acceleration are examples of vector quantities. Scalar quantities can be added, subtracted, multiplied, and divided directly. Vector quantities, because of the incorporation of a second dimension (direction), must be treated differently.

VECTOR NOTATION

A vector quantity (figure 4-1) can be represented by a line segment. The length of the line represents the magnitude of the quantity, while the angular position of the line segment and an arrow point indicate the quantity's direction.

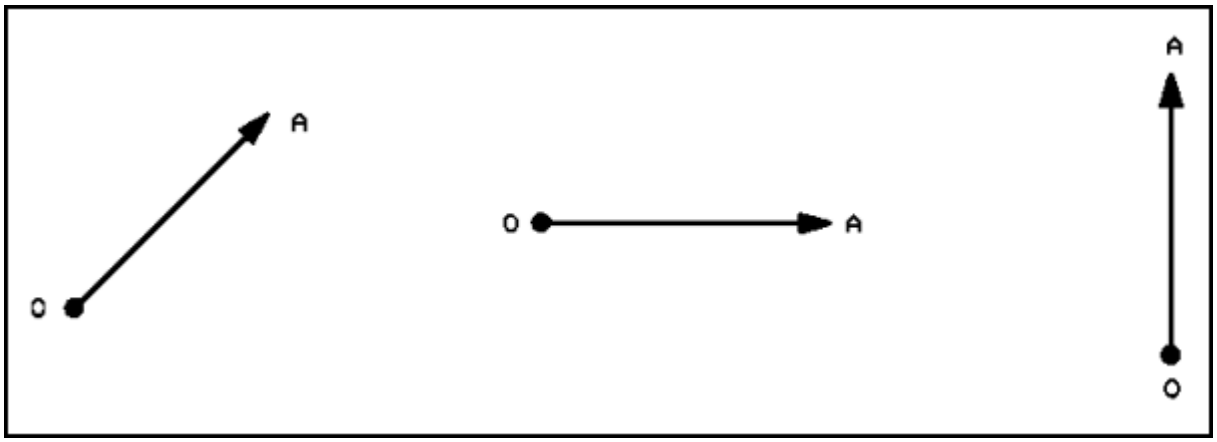

Figure 4-1. Vector Quantity.

Usually, this quantity is known as vector OA; however, it may be written $\overline{OA}$ or OA. Occasionally, it may be designated by a single letter such as $\overline{M}$, or, or $\dot{M}$, or M.

RESULTANT VECTOR

Generally, vectors are combined as indicated in figure 4-2. Since the two vectors $\overline{OA}$ and $\overline{BC}$ have the same direction, they can be combined to produce one single vector $\overline{OC}$ (figure 4-3), having a magnitude of 21 units and keeping the original direction.

Figure 4-2. Two Vectors of the Same Direction.

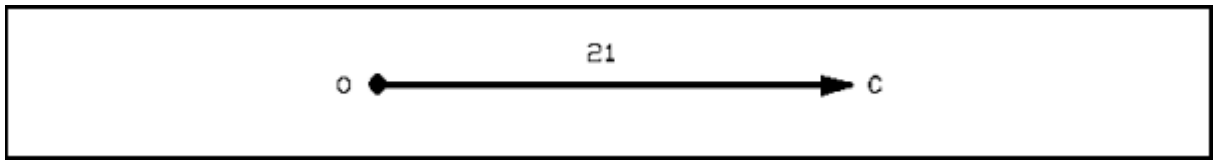

Figure 4-3. Two Vectors of the Same Direction Combined.

Resolving the principle shown in figure 4-3 into a practical illustration, suppose a boat is traveling due east at 10 mph, and a current is flowing due east at 11 miles per hour. The result will be a speed of 21 miles per hour for the boat in due east direction. Conversely, if the vector $\overline{BC}$ were in the opposite direction figure 4-4), the result would be the new vector $\overline{OC}$ (figure 4-5). It would have a magnitude of 1 and be in the direction as illustrated.

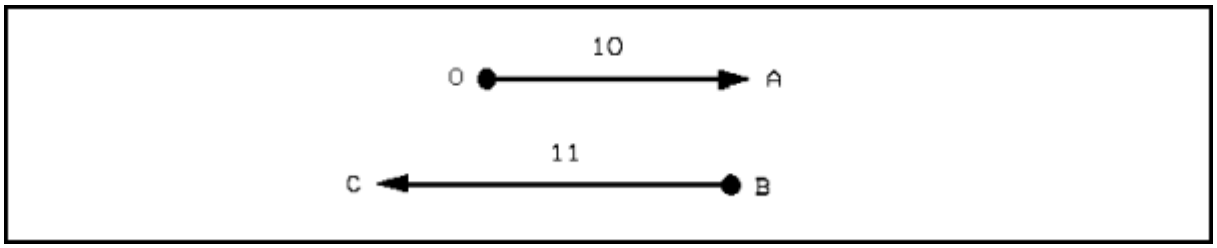

Figure 4-4. Vectors of Opposite Directions.

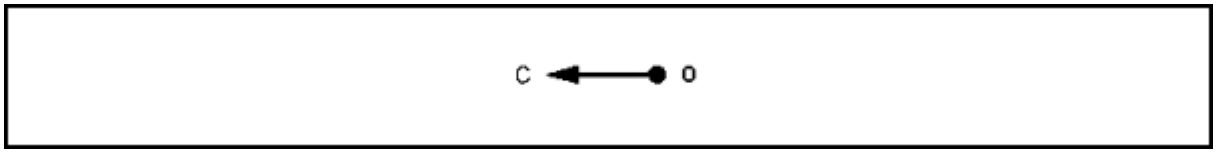

Figure 4-5. Vectors of Opposite Directions Combined.

Two vectors not in the same direction can be combined using the parallelogram method. By placing the heels of the vectors together and completing a parallelogram (figure 4-6), the diagonal of this parallelogram represents the resultant vector. For example, combining vector $\overline{OA}$ and vector $\overline{OB}$ and completing the parallelogram, you get $\overline{OC}$ as the resultant vector.

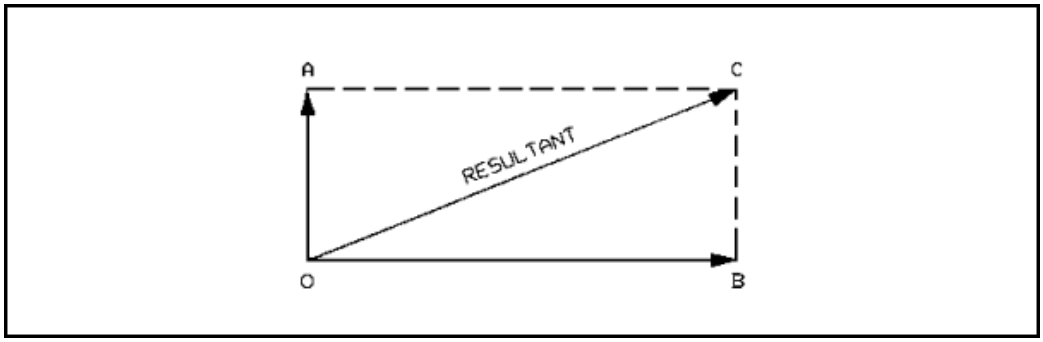

Figure 4-6. Parallelogram Method of Combining Vectors.

Three or more vectors can be combined in much the same way. By keeping their same relative directions and placing them "heel to toe," the resultant vector is the vector joining the heel of the first vector to the toe of the last vector. For example, the following three vectors, $\overline{OA}$, $\overline{PQ}$, and $\overline{MN}$ (figure 4-7) are combined to produce the resultant vector $\overline{ON}$. Combining the vectors, you get the vector in figure 4-8.

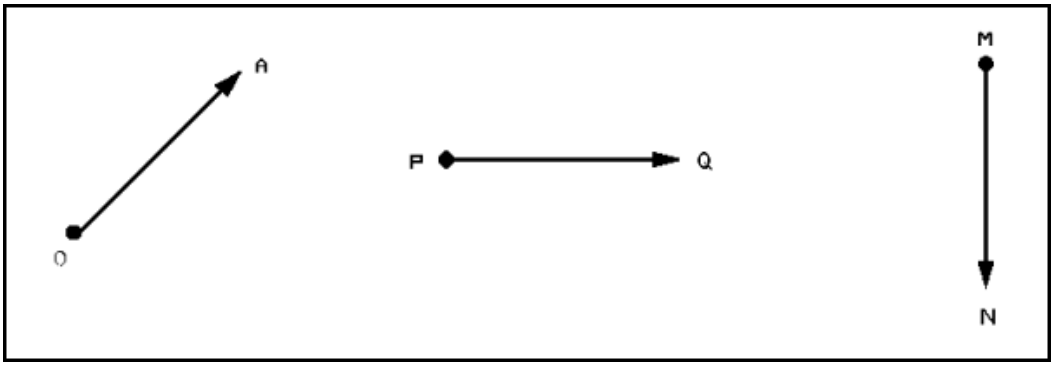

Figure 4-7. Three Vectors To Be Combined.

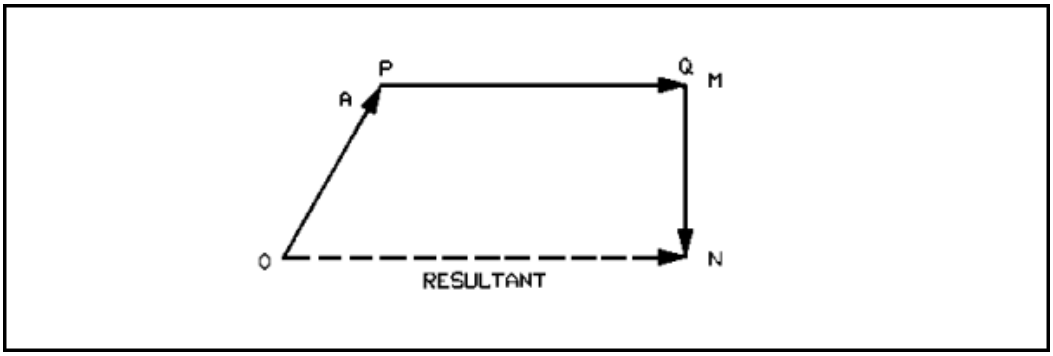

Figure 4-8. Parallelogram Method To Find the Resultant Vector.

VECTOR REPRESENTATION

Vectors are usually described in polar or rectangular form. The polar form shows the magnitude of the vector and the angle that it makes with the horizontal. For example, 10/30° describes a vector 10 units long at an angle of 30°. In rectangular form, the vector is resolved into its horizontal and vertical components, which are its projections on the horizontal (X) and vertical (Y) axes, and which have as their origin, the origin of the original vector. Thus, for vector $\overline{OA}$ (figure 4-9), the vertical component can be expressed

$Y = \overline{OA} \sin \theta$,

and the horizontal component can be expressed

$X = \overline{OA} \cos \theta$.

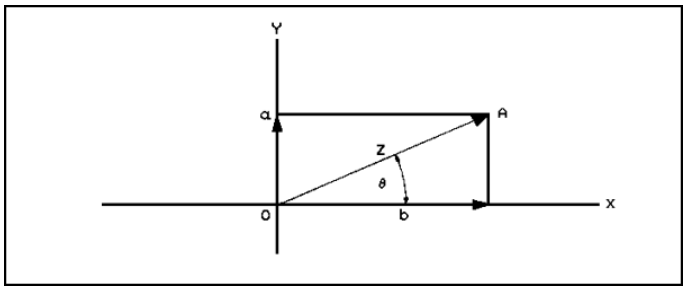

Figure 4-9. Polar Form of Representing Vectors.

Conversely, the original vector $\overline{OA}$ could be determined from the two component vectors X and Y (figure 4-10).

Assume:

X and Y are given (figure 4-10).

θ and magnitude of $\overline{OA}$ is unknown.

Therefore,

$\tan \theta = \frac{Y}{X}$-, and

the magnitude of $\overline{OA} = \sqrt{-X^2 + .Y^2}$.

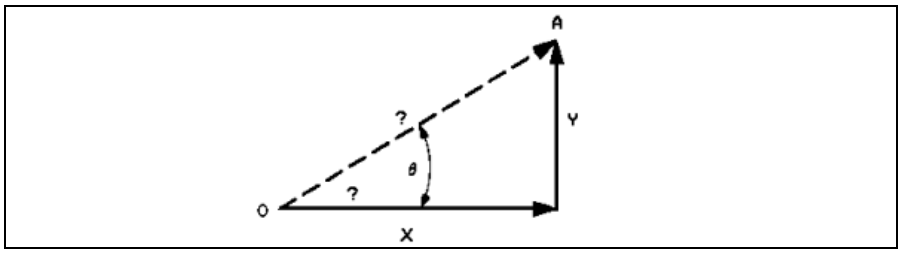

Figure 4-10. Determining Vector in Polar Form.

When dealing with the electricity, you usually have to express a vector in terms of rectangular coordinates; however, the vertical axis of the coordinate system is designated as the imaginary axis (j component), while the horizontal axis is designated the real axis. In rewriting vector 10/$\underline{30°}$ in terms of its real and imaginary components, consider first the following illustration (figure 4-11).

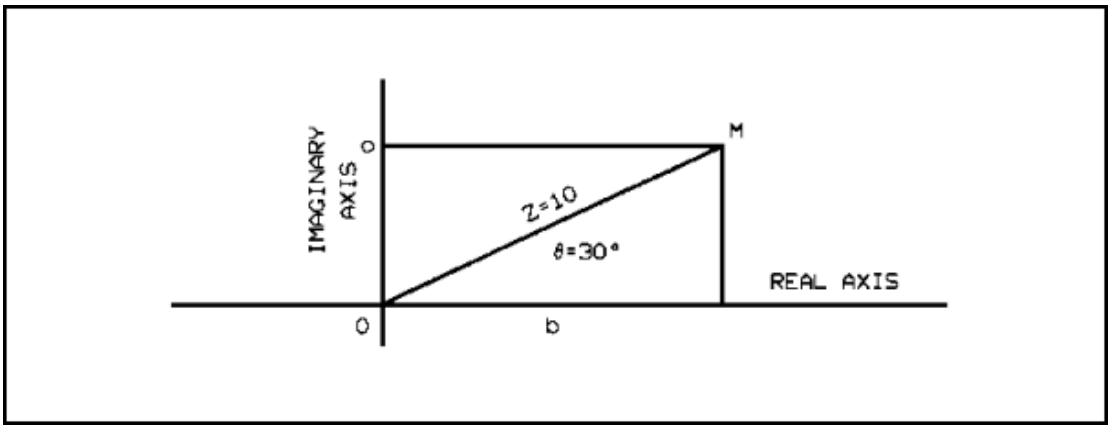

Figure 4-11. Rectangular Form of Representing Vectors.

By stating that 10/$\underline{30°}$ could be represented in terms of vertical and horizontal components, you have essentially stated that

$$10/\underline{30°} = b + ja.$$

However,

$$b = Z \cos \theta,$$

and

$$a = Z \sin \theta.$$

Therefore,

$$\begin{aligned} 10/\underline{30°} &= (10 \cos \theta) + j(10 \sin \theta) \\ &= (10 \cos 30°) + j(10 \sin 30°) \\ &= (10 \times 0.866) + j(10 \times 0.5) \\ &= 8.66 + j5. \end{aligned}$$

CALCULATIONS

Addition and Subtraction

Since the addition and subtraction of vectors by graphic means is not sufficiently accurate without a precise measuring instrument, the usual practice is to convert vectors to their rectangular form and then to add or subtract them algebraically.

Problem:

Add 35/40° and 47/55°

Solution:

$35/\underline{40°} = 35\cos 40° + j35 \sin 40°$

$= 35 (.7660) + j35 (.6428)$

$= 26.81 + j22.50$

$47/\underline{55°} = 47 \cos 55° + j47 \sin 55°$

$= 47(.5736) + j47 (.8192)$

$= 26.96 + j38.50$

$$\begin{array}{l} 26.81 + j22.50 \\ \underline{26.96 + j38.50} \\ 53.77 + j61.00 \end{array}$$

To convert to polar form, remember that

$$\tan \theta = \frac{\text{scalar value of the vertical component}}{\text{scalar value of the horizontal component}}$$

$$= \frac{61.00}{53.77}$$

$= 1.134.$

Therefore,

$\theta = 48.6°.$

Remembering that the horizontal component (b) was equal to Z to cos θ, we can solve the value of Z.

In polar form, the new vector representing the sum of $35/\underline{40°}$ and $47/\underline{55°}$ is $81.3/\underline{48.6°}$.

Multiplication

To multiply two vectors in polar form, multiply the magnitudes together and add the angles.

$55/\underline{40°} \times 47/\underline{55°} = (55 \times 47) / \underline{40° + 55°}$

$= 2{,}585/\underline{95°}$

You can get the same result by converting the vectors to rectangular form, performing the multiplication, and converting the product back to polar form.

Division

In dividing vectors, divide the magnitudes and subtract the angles.

$60/\underline{40°} \div 30/\underline{20°} = \frac{60}{30}/40° - /\underline{20°} = 2/\underline{20°}$

RAISING A VECTOR TO A POWER

Since a power of a quantity is essentially a repeated multiplication process, $(15/20°)^3$ is essentially

$(15/20°)\ (15/20°)\ (15/20°)$,

which revolves into

$(15^3)/20 + 20 + 20 = 3{,}375\ /\underline{60°}$.

ROOT OF A VECTOR

To extract the root of a vector in polar form, extract the root of the magnitude and divide the angle by the degree of the root. For example,

$\sqrt[3]{8/60°}$

can be rewritten as

$\sqrt[3]{8/60°} \div 3 = 2/\underline{20°}$.

Lesson 4 Review Exercises

REVIEW EXERCISES

1. Which of the following is the correct polar form for a vector whose magnitude is 10 at an angle of 30°?

   a. 8.61 + j5.

   b. 30 + j10.

   c. 30°/10.

   d. 10/30°.

2. When multiplying two vectors in polar form, what must you do to the angles?

   a. Add.

   b. Subtract.

   c. Multiply.

   d. Divide.

3. Which of the following do vector quantities have?

   a. Magnitude.

   b. Direction.

   c. Magnitude and Direction.

   d. Neither.

4. What method is used to combine two vectors not in the same direction?

   a. Parallelogram.

   b. Square.

   c. Rectangle.

   d. Trapezoid.

5. Which form shows the vector's magnitude and the angle it makes with the horizontal?

   a. Rectangular.

   b. Polar.

Recheck your answers to the Review Exercises. When you are satisfied that you have answered every question to the best of your ability, check your answers against the Exercise Solutions. If you missed two or more questions, you should retake the entire lesson, paying particular attention to the areas in which your answers were incorrect.

Lesson 4 Review Exercises Solutions

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Lesson 4